Computer Applications to Tides in the National Ocean Survey

Supplement to Manual of Harmonic Analysis and Prediction of Tides (Special Publication No. 98)

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U.S. DEPARTMENT OF COMMERCE
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Supplement to Manual of Harmonic Analysis and Prediction of Tides (Special Publication No. 98)

by

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CONTENTS

1. INTRODUCTION

The state of the art in tide analysis and prediction reached levels of achievement in the early part of the 20th century that were not significantly improved upon for about 50 years. Even the early electronic computers (IBM 650 and 1620 were the first acquired by the U.S. Coast and Geodetic Survey) were unable to outperform the mechanical tide prediction machine completed by R. A. Harris and E. G. Fischer in 1910. However, the advent in the early 1960's of new, faster electronic computers finally brought about the replacement of the various classical tide analysis and predicting procedures.

That the predicting machine resisted so well and so long its eventual replacement, is a tribute to the skill and ingenuity of its creators. The machine had the ability to sum continuously with great precision 37 incommensurable cosine curves of periods ranging from 8 cycles per lunar day to 1 cycle per lunar year, draw a continuous plot, and select the times and heights of extremes (high and low waters). In the most recent years, it produced automatically a typed manuscript ready for reproduction and inclusion in the tide tables. Those of us involved in replacing this machine had severely mixed emotions for it was a remarkable mechanical achievement worthy of respect. Nevertheless, it was a relief to know that we were no longer dependent for tide predictions on one single set of hardware that would eventually wear out or, even worse, could be terminated by some natural or man-made catastrophe.

During a period of conversion to computer use, there is a temptation to have an electronic computer do exactly what man has done before and thereby strive only to reduce the human effort. To a large extent this was resisted in the tides program, and numerous changes in procedures were designed to optimize the potential of large, fast computers rather than routinely follow old procedures. For this reason, many of the procedures described in National Ocean Survey (previously U.S. Coast and Geodetic Survey) tide manuals are obsolete. This publication is designed to serve as an interim substitute for necessary revisions of these manuals.

Methods of tide analysis have changed greatly. Some processes have been completely revolutionized while others are fundamentally unchanged except for the substitution of computers for human effort. The following text describes these changes in both prediction and analysis procedures with reference to appropriate paragraphs in various tide manuals.

In the discussion that follows, the terms "species" and "group" will be used to identify sets of tidal constituents; these terms are used in the historic harmonic development of the tide-producing potential by Doodson (1921) and, more recently, by Cartwright and Tayler (1971). Any tidal constituent has a frequency equal to an integral sum and/or difference of six fundamental frequencies corresponding to the following periods:

(a) 1 day (period of Earth's rotation relative to Sun)

(b) 1 month (period of Moon's orbital motion around Earth)

(c) 1 year (period of Earth-Moon orbital motion around Sun)

(d) 8.85 years (period of lunar perigee)

(e) 18.61 years (period of regression of lunar nodes)

(f) 20,900 years (period of solar perigee)

Using Doodson notation for combinations of these six frequencies, M_2 and S_2 are identified by 255.555 and 273.555 respectively (200000 and 22-2000 in

Cartwright and Tayler notation); the difference in notation is that Doodson uses "5" for zero for the last five frequency categories to avoid negatives, whereas Cartwright and Tayler base on zero and then show negatives when necessary. In either system, all constituents having the same digit in the first column are in the same species, i.e., the same number of cycles per day. All constitutents having the same first two digits are in the same group. Thus M_2 and S_2 are in the same species but not in the same group.

Perhaps at some future time, consideration will be given to adding new constituents in lieu of the node factors and phase corrections presently used. For example, Cartwright and Tayler (1971) constituent 2000-10 (one cycle per 18.61 years different than M_2 and 3.7 percent of its amplitude) would be an improvement over the M_2 f and u presently used for the middle of a year being predicted.

The state of the art in tide prediction is described in greater detail in an essay by this author in the volume Geophysical Predictions published by the National Academy of Sciences, 1978; the article is Appendix B in this report.

Whenever the title "Coast and Geodetic Survey" is used in the various manuals, it should be replaced by "National Ocean Survey". Various comments and criticisms by numerous oceanographers at National Ocean Survey (NOS) are acknowledged with thanks. Inasmuch as not all suggestions were followed, I accept responsibility for both format and contents.

2. REVIEW OF NATIONAL OCEAN SURVEY MANUALS

(a) Manual of Harmonic Analysis and Prediction of Tides
C&GS Special Publication No. 98, Revised (1940) Edition
Paul Schureman, 1941

The fundamental principles described in this manual are unchanged. Scientists have more recently achieved some refinements in astronomical constants listed in Table 1 of the manual; a comprehensive revision of the manual should carry these into the various calculations of the equilibrium (theoretical) tide. This is not done here, nor is it anticipated that significant changes in tide predictions will result from such a modification in the future. The changes to be described in the following text are those that have already been made in National Ocean Survey (NOS) computing procedures.

Tide analysis: The traditional principles of analysis are unchanged, i.e., one solves for the amplitudes and phase lags of a finite set of tidal constituents established as contributing to tides (from equilibrium theory and from nonlinear interactions of principal equilibrium constituents). This may be done (1) by the classical procedure of solving for one constituent at a time and then removing the effect (sidebands) of constituents nearby in frequency (same number of cycles per day) by "elimination" procedures and (2) by a least, square solution of normal equations, derived from observational equations, to solve for all constituents simultaneously. The latter procedure, not described in S.P. 98, was extended to a much larger set of constituents (many added nonlinear combinations called compound tides, shallow water constituents, and overtides) by Zetler and Cummings (1967) to improve tide predictions at Anchorage, Alaska.

This process, developed independently in England by Rossiter and Lennon (1968), is called extended harmonic analysis. In the Zetler and Cummings analysis (114 tidal constituents), it was found feasible to resolve various species separately so long as all constituents within a species are solved for simultaneously. This is comparable to classical analysis where elimination of sideband effects is concerned only with contributions from other constituents in the same species.

S.P. 98 describes only the first procedure; the second requires solving large matrices, impossible until the more recent advent of large computers. A fundamentally different approach, response analysis and prediction (Munk and Cartwright, 1966) is described in the comments for par. 341. Empirical comparisons between response and harmonic predictions (Unesco, 1975, and a 1979 test organized by NOS) indicate slightly superior results by the response method. However, it is not yet clear whether the small improvement justifies substituting the response method for the classical harmonic methods.

Tide prediction: The mathematical formulation is unchanged, but the predictions are prepared using electronic computers rather than a mechanical analog machine. Times of high and low waters are determined by successive approximation rather than by selecting times when the sum of the derivatives equals zero. In addition to the greater efficiency implicit in electronic computers, a restriction to a finite set of tidal constituents (those on the predicting machine) has been removed, and the risk of depending on a particular machine has been eliminated.

The comments that follow use the paragraph numbers in S.P. 98 for identification purposes:

INTRODUCTION

1-29: No changes.

DEVELOPMENT OF TIDE-PRODUCING FORCE

30-141: No changes, except, as in par. 72, references to "constituents used in the Coast and Geodetic Survey tide-predictions machine" should be interpreted as constituents NOS normally uses. This also applies to similar notes in subsequent paragraphs.

ANAYLSIS OF OBSERVATIONS - HARMONIC CONSTANTS

142-146: No changes.

147: Add as follows: If the record breaks are significant and both interpolation and breaking the record into shorter series are undesirable, consideration should be given to using least-squares analysis (see notes for par. 155-186 for more details).

148-149: No changes.

150-153: Add the following: The recommendation to use approximations to synodic periods of the principal constituents is clearly valid for analyses in which each constituent is first resolved separately, since the contribution to the first approximation due to interfering constituents is minimized (i.e., corrections during elimination procedures are minimized, and therefore, optimum accuracy is achieved).

Whether synodic periods are necessary or even preferable for least-squares solutions (i.e., all constituents resolved in a single matrix) is not as clear cut. It appears to be obvious that two frequencies can be resolved by least squares fitting in much less than a synodic period in the absence of noise (Munk and Hasselmann, 1964). However, with real data (hence a noise continuum) it appears possible that an analysis may be improved by choosing a record length approximating the synodic periods of the principal constituents. This has not been demonstrated, but it seems prudent to conform by using appropriate synodic periods until the question has been adequately resolved.

154: No changes.

155-186: Delete and substitute: Stencil summing as a means of computing mean values for constituent hours is a procedure to approximate what would be achieved by cutting the marigram (tide record) into constituent periods (for example, every 25.82 hours for O_1), piling the cut sections vertically and averaging a mean curve for the constituent period through the pile. It is, of course, not feasible to cut the record repeatedly for different constituent periods, and it is impossible to sum vertically through the cut sections. By means of stencil overlays (described in these paragraphs), hourly heights are identified as constituent hours (with errors not exceeding 0.5 hour and algebraically averaging near zero); then a Fourier analysis is calculated for the constituent hourly means and, in some cases, for some harmonics of the constituent frequency. An augmenting factor is applied to the amplitude to correct for the small reduction due to the use of solar rather than constituent hours.

The above procedure is no longer used. For computerized 14-, 15-, and 29-day analyses, a special form of Fourier analysis is computed for each constituent frequency:

$$C_\omega = \frac{2}{n}\sum_{i=0}^{n-1} h_i \cos ia_\omega$$

$$S_\omega = \frac{2}{n}\sum_{i=0}^{n-1} h_i \sin ia_\omega$$

where C_ω and S_ω are real (in phase) and imaginary (out of phase) amplitudes, n is the number of hourly values, h_i are the hourly heights (i=0 to n-1), and a_ω is the constituent speed in degrees per solar hour. As before,

the phase $$\zeta_\omega = \tan^{-1}\frac{S_\omega}{C_\omega} \qquad \text{(equation 303)}$$

and the amplitude $$A_\omega = (C_\omega^2 + S_\omega^2)^{\frac{1}{2}} \qquad \text{(equation 304)}$$

The above is a special form of Fourier analysis because ordinarily the constituent period is not an exact harmonic of the length of record being analyzed. The program in use is designed also to apply to series equally spaced in time with time intervals different than 1 solar hour and for computing Fourier coefficients for harmonics of constituent frequencies. The above calculation substitutes for Form 194 in the harmonic analysis procedure; the augmenting factor previously used is unnecessary in the revised procedure, described in greater detail by Dennis and Long (1971). Form 194 is still in use for calculating S_a and S_{sa} although this could readily be done in a computer Fourier analysis.

Least squares analysis solves for all constituents simultaneously and does not use Fourier techniques. Each observed height is made an observation equation, $h_i = H_0 + \Sigma C_\omega \cos a_\omega t_i + \Sigma S_\omega \sin a_\omega t_i$ for n observed heights (h_i) at times (t_i) for i = 0 to (n-1), m constituents from ω = 1 to m and a constant, H_0. As above, C_ω and S_ω are the real (in phase) and imaginary (out of phase) amplitudes, and a_ω is the constituent speed in degrees per hour. Comparable expressions can be used with sampling intervals different than an hour. Solving for (2m + 1) unknowns in the least squares sense furnishes (2m + 1) normal equations that can be resolved by matrix algebra (but impossible before large electronic computers). The mathematics is unchanged if some h_i's are missing, but a change in procedure is necessary. If there is one or several breaks in a record, the program identifies the time of each resumption of observations so that the phases of the added section are referred to the original start time; there are fewer observation equations because of the loss (or losses) of record, but otherwise the process is unchanged. If there are many breaks or the data are random in time, the time of each observation must be identified; a procedure for harmonic analysis of random data is described by Zetler et al. (1965). The constituent phases and amplitudes are computed for each constituent as before in equations 303 and 304. With least squares analysis, the elimination procedure is not necessary.

187-209: These paragraphs are generalized discussions of Fourier analysis. They are no longer pertinent since mean constituent curves are not computed in any tidal analysis procedure NOS now uses. However, formulas 303 and 304 are used in the Fourier analyses procedures in the notes for paragraphs 155-186.

210-220: These refer to mean values obtained from stencils of hourly (solar) heights, and therefore, they are not applicable to presently used procedures.

221-228: No changes.

229-243: The procedures described for inference assume that the tidal admittances (Munk and Cartwright, 1966) are linear within a species. The assumption is not necessarily valid but is probably as good as can be done if no additional information is available (such as amplitude and phase relationships well determined at a nearby location). However, see the comments for paragraph 341 concerning a reduced accuracy in inferences which use S_2 amplitudes and/or phases (because traditionally analyzed S_2 includes both gravitational and radiational contributions).

For a long time it has been the custom in NOS analyses to reject analyzed amplitudes less than 0.03 foot and to substitute inferred values for these on the basis that, for such small values, analyzed phases are unreliable. This procedure was implemented by Schureman on empirical evidence from successive analyses for San Francisco tides. It has more recently been documented in a number of studies that the continum (noise level) is frequency-dependent, highest in very low frequencies and decreasing monotonically with higher frequency. Accordingly, a small amplitude that should be rejected for the monthly or fortnightly tides may be useful at 1 or 2 cycles per day; tests on this point are presently underway.

244-253: The elimination procedures shown are still used in analyses of 29-, 15-, and 14-day series; they are done routinely in computer programs. They are not necessary in those least squares analyses in which all constituents are resolved simultaneously. For least squares analysis of records less than a year in length, an approximation procedure for refining the harmonic constants for the resolved constituents for the influence of nearby unresolved constituents is described in Zetler et al. (1965).

254-285: The annual and semi-annual constituents (S_a and S_{sa}) are important in tide predictions, and these can be reasonably approximated from analysis of monthly means as indicated. Inasmuch as these tides are primarily radiational (steric heating of near-surface water and/or seasonal wind effects), they will vary significantly from year to year and therefore the longest available series (i.e. the mean of all Januarys, the mean of all Februarys, etc.) are used for these calculations. As noted, the fortnightly and monthly tides are not ordinarily important and usually results from analyses are unreliable due to the high level of the continuum at these frequencies. If they are to be determined, their amplitudes should be compared with the nearby continuum to measure their significance. One method to do this using electronic computers is described by Zetler (1964). After the species 1 to 8 tides have been removed by a band-pass filter and/or subtracting a tide prediction, the data are lowpassed, decimated, and truncated so that the frequency sought is very close to a harmonic of the series length. A Fourier analysis of roughly 10 harmonics on either side of the constituent frequency will reveal the continuum level for comparison with the constituent amplitude. A different truncation will be needed for each constituent (Mm, Mf, and Msf) to place its frequency near that of a harmonic for the series length. Figure 1 is a plot of results of such a calculation (Zetler, 1964).

286-300: These procedures are seldom if ever used. In principle, a least squares solution is feasible for the tidal constituents using the four values per day.

301-328: Some of the procedures illustrated for analysis of tides are done away with completely; others are programmed for calculation on computers. The hourly height form (no. 362, fig. 9) is no longer needed in this format and has been replaced by a form showing hourly heights for a full month and by a computer printout similar to form 362 with up to 11 days per page. Stencil sum illustrations (fig. 10 and 11) and the stencil sum form (no. 142, fig. 12 and 13) are obsolete and not replaced. Forms 244 and 244a, "Computation of $V_0 + u$" and "Log F and Arguments for Elimination," (fig. 14 and 15) are ordinarily calculated within computer programs. Form 194, "Harmonic Analysis" (fig. 16) is ordinarily replaced by computer Fourier calculations for specific frequencies; the augmenting factor is no longer necessary. The computations for inference and elimination (forms 452 and 245, fig. 17, 18, and 19) are done by the computer for 15- and 29-day analyses; they are unnecessary in least squares analyses for a year of data.

329-340: The procedure for analysis of tidal currents are still valid but Fourier calculations should be made to obtain the amplitudes and phases to be used on form 723 (no stencil summing or form 194 calculations). The Fourier calculations can be made with half-hourly data, as well as for any other equally spaced time interval for the observations.

341: The comments relative to the use of a predicting machine for a harmonic prediction are equally valid with respect to using an electronic computer. Replace Coast and Geodetic Survey with National Ocean Survey.

A third method of tide anlaysis and prediction (Munk and Cartwright, 1966) uses the following formalism:

For any linear system, an input function $x_m(t)$ and an output function $x_n(t)$ are related according to

$$x_n(t) = \int_{-\infty}^{\infty} x_m(t-\tau) w_{mn}(\tau) d\tau + \text{noise}(t)$$

where $w_{mn}(\tau)$ is the "impulse response" of the system, and its Fourier transform

$$Z_{mn}(f) = \int_{-\infty}^{\infty} w_{mn}(\tau)\ e^{-2\pi i f \tau} d\tau = R_{mn}(f)\ e^{i\phi_{mn}(f)}$$

is the system admittance (coherent output/input) at frequency f.

Response tidal analysis and prediction uses as input functions the time-variable spherical harmonics of the graviational potential and of radiant flux[1] on the Earth's surface. In practice, the integrals are replaced by

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[1] A function designed to vary with the radiant energy falling on a unit surface in a unit time; it is related to daily atmospheric pressure and wind variations and to seasonal changes in ocean temperature.

summations; x_m, w, and Z are complex. The discrete set of w values are termed response weights.

This method is the first successful major departure from the traditional solutions in which the tide oscillations are described by the amplitudes and phase lags for a finite set of predetermined frequencies. Subsequently it was recommended that response analysis of short records (about 1 month) of pelagic tidal measurements use a response prediction at a nearby coastal station as the reference series (Cartwright et al., 1969). The calculation of traditional harmonic constants from response admittances made results of the two methods compatible (Zetler et al., 1969). The optimum number of weights in response analysis depends directly on the length of record and inversely on noise level in a tidal band; more weights degrade the prediction and generate an artificial wiggliness in the admittance (Zetler and Munk, 1975). This study showed for the first time that better results may be obtained by centering the lags to a potential retarded by the age of the tide[2] rather than to a potential centered on the prediction time.

SCOR[3] Working Group No. 27, "Tides of the Open Sea," conducted an analysis workshop in conjunction with an intercomparison of open sea tidal pressure sensors. The report (Unesco, 1975) shows a clear superiority for response

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[2]The time interval between a maximum range in the equilibrium tide and a comparable range at a particular place. For example, equilibrium spring tides occur at new and full moon; in the ocean they occur 0.984 ($S_s{}^\circ$-$M_2{}^\circ$) hours later. There are comparable ages for maximum ranges related to perigee and to maximum declination of the moon.

[3]Scientific Committee on Ocean Research.

procedures as compared with classical methods used by various national tidal groups. In addition to this statistical advantage, response analysis is more intellectually pleasing in that one uses the entire tide-producing potential rather than having to arbitrarily choose a finite set of tidal frequencies.

The ability to separate gravitational and radiational contributions to the tide resolves an unsatisfactory aspect of results from classical analysis. If one plots the phase angles or the amplitude admittances (ratio of analyzed to equilibrium amplitudes) from traditional analysis against frequency, one usually finds a sharp bend or even a discontinuity at S_2 (30°/solar hour). It is not reasonable that the oceans should exhibit such abrupt changes, particularly at always the same frequency. The smooth plots obtained for the gravitational admittances using response procedures are undoubtedly due to the radiational inputs having been separated from the gravitational whereas classical analysis produces vector sums of the two.

Cartwright (1968) and Zetler (1971) found by different empirical approaches that the radiational S_2 has an average amplitude of roughly one-sixth the gravitational S_2 and that the phases of the two are quite different. Inferences of constituents nearby to S_2 in frequency that use the S_2 values from traditional harmonic analysis (par. 229-243) may be significantly less accurate because of the contribution of radiational tides to the S_2 harmonic constants.

342: Unchanged:

343: A predicting machine finds the times of high and low tides by determining mechanically the times when the sum of the first derivatives is equal to zero (equation 452). Modern electronic computers are so fast that they can use successive approximations of the sum of the cosine curves (equation 451) to calculate times and heights of high and low water.

344-352: Unchanged, except as already noted for paragraph 343.

353-420: The descriptions furnished are still valid but in a historical sense only, since the mechanical tide predicting machine is no longer used by the National Ocean Survey. In its last years of use, a read-out device had been added to the tide predicting machine so that the tide tables were prepared automatically by a special typewriter in a format for direct reproduction in the tide tables. To facilitate this conversion, times of tide were printed in a 24-hour system, whereas previously light and dark type had been used for a.m. and p.m. times respectively. All tide predictions since 1965 have been prepared on electronic computers; the programs are designed to prepare the tide and tidal current tables in a format for direct reproduction. There no longer is a constraint restricting the user to the 37 constituents described in the manual; Zetler and Cummings (1967) used 114 constituents with frequencies up to 12 cycles per day to improve tide predictions for Anchorage, Alaska.

421-432: The procedures outlined are for the tide predicting machine. Comparable calculation, as necessary, are included in the computer programs. It is not necessary to check values at later dates in the year (form 445).

433-447: As with tides, predictions for reversing and hydraulic currents are readily prepared on electronic computers, substituting the permanent current for the datum constant used with tides. Rotary tidal current predictions, not possible on a mechanical tide predicting machine, are now being prepared on electronic computers.

TABLES

Table 1: Refinements of fundamental astronomic data have been accomplished since the publication date of S.P. 98. In 1964 the International Astronomical Union adopted the following:

Equatorial radius of earth	6,378,160 M.
Ratio of mass of earth to mass of moon	81.30
Ratio of mass of sun to mass of earth	332,958

Note that S.P. 98 lists the ratio of the mass of sun to combined mass of earth and moon.

Solar parallax	8.79,405"
Lunar parallax	3,422.451"

Until a complete revision of S.P. 98 is prepared, it would be desirable that the best determined astronomical data be incorporated into the fundamental tidal computations. Nevertheless, preliminary tests have shown that these changes would not significantly improve routine tidal predictions.

Table 2: Unchanged:

Table 2a: May be greatly expanded as in Zetler and Cummings (1964) and in published lists of tidal constituents for some foreign ports.

Tables 3-6: Unchanged.

Table 7: Logs are not used in calculations.

Table 8: Unchanged.

Table 9: Logs not used.

Tables 10-11: Unchanged.

Tables 12-13: Logs not used.

Tables 14-18: Unchanged.

Tables 19-20: Not used.

Tables 21-26: Unchanged, but one may calculate these rather than use a look-up procedure.

Tables 27-28: Logs not used.

Table 29: Used, as applicable, in computer program.

Tables 30-34: Not used.

Table 35: In computer program but also useful for calculations such as equation 455.

Tables 36-38: Not used.

Tables 39-42: Unchanged.

(b) Manual of Tide Observations

Coast and Geodetic Survey Publication 30-1 (1965)

The anaolg-to-digital recorder tide gage is described briefly in the addenda (par. 260-271) of this publication; the automated processing of the digital data is not included. The latter, as described here, modifies the description for tabulation and reduction in par. 212-245.

As noted in par. 260, the analog-to-digital recorder tide gage provides a punched-tape record of the water level, to the nearest 0.01 foot, at selected intervals, usually 6 minutes (0.1 hour). After mechanical translation to punched cards and/or magnetic tape, editing procedures check time and date sequences on successive cards and output a diagnostic for incorrect sequence or missing values; the format of each card is checked routinely. The heights are edited by computing third differences which, on a smooth curve, tend to be near zero. An anomaly in the original data is tripled in the third differences, thereby, permitting a simple diagnostic for human review of the data when calculated third differences exceed a preset limit. This editing program is less likely to reveal a situation in which the float is hung up in a fixed position for some time, making it necessary for other tests to disclose this situation. A program was written to prepare a histogram of the distribution of the last digit (how many end in 0, how many end in 1, etc.) but this is not used routinely.

Par. 212-217 describes high and low water tabulations, now done automatically from water level values for each 0.1 hour. Just as the selection of times and heights of high and low waters from a continuous plot on a marigram requires some visual smoothing of the record, a similar automated process is necessary with digital data. This differs from the preparation of predicted tide tables with adjacent predicted heights routinely compared to select the extremes (high and low waters). Because of the presence of noise in the observations, consideration was given to using a low pass filter; this was rejected on the basis that it modifies the data on a subjective basis (the choice of filter), and it is not possible now to anticipate future uses of the data that might be affected by the choice of the filter. Furthermore, there is the problem that the records are frequently used as legal records, and a smoothed value might have a questionable value in a court of law. In addition, the response of the smoothing function must be exactly unity in the tidal frequencies; otherwise the analyzed ranges would need to be corrected. It appeared logical and simple to identify a limited period of an extreme and then to fit a parabola by least squares to these data. The first derivative of $Y = ax^2 + bx + c$, in which a, b, and c are the unknowns to be obtained by a least square procedure, can be set equal to zero ($2ax + b = 0$) to obtain a linear solution for the times of high or low water. However, this implies a model curve symmetric to a line parallel to the Y axis, a solution that is not valid for the skewed curve found in many estuaries where the duration of rise is shorter than the duration of fall. Instead, a tilting board type of approach is used with a given time as a fulcrum, adding the four adjacent heights on either side and subtracting one sum from the other. At times of extremes, this difference is closest to zero. The method stemmed from an approach of fitting a line to the same data by a regression formula and choosing the slope closest to zero, but the latter method weights each height by its distance from the fulcrum, making the distant points most important. The method accepted gives equal weight to each height used in the computation.

Times are treated cumulatively during a month, and the program sums these times for both high and low waters. The sum of the moon's transits for the month has already been prepared on the same basis using a formula which takes into account the dates of the 2 days with only one transit due to the lunar progression in solar time. The instruction card which precedes the data contains this sum of transits as well as the number of days in the month. Also included is the designated option of reducing the data as (1) semidiurnal, (2) mixed, and (3) diurnal (only one high and low water during some days of the month).

With the first option, the output lists times and heights of all high and low waters, mean lunitidal intervals, mean high water, mean low water, mean tide level, mean sea level (river level in estuaries), mean range, hourly heights (these are outputted on punched cards as well), and monthly extremes. Option two chooses the higher high and lower low water each day and adds mean higher high water, mean lower low water, and the mean diurnal inequalities to the output. With both of these options, the program demands the proper number of high and low waters (two less than twice the number of days). The program does not sum the last tide in a month if that tide is superfluous, and it advances to the early hours of the first day of the following month if another tide is needed. A diagnostic is outputted if there are too many or too few calculated high and low waters in a given month. The diagnostic includes information identifying the problem dates.

With the third option (sometimes diurnal), the program calculates the times and heights of all high and low waters and outputs them according to the day of the month. This option also outputs and punches hourly heights and lists monthly extreme high and low. It does not reduce the data nor does it check the number of highs and lows.

The procedures used in the program to locate times of extremes makes it possible to identify other anomalous tide conditions. Inasmuch as it is impossible to anticipate and correct for every possible contingency, the program outputs diagnostics giving the approximate time of the occurrence and categories of what may be wrong. Thus, if a float sticks to the side of the well, the resulting straight line is detected and reported. If a storm surge obliterates a high or low tide, the failure to change sequence or the wrong number of tides is reported. Under these circumstances, someone must examine the data and decide on how the processing must proceed. Simple points of inflection or stands are handled within the program.

Par. 218-220 describes hourly height tabulations. As noted above, the automated procedures for high and low water tabluations also provide an hourly height computer printout similar to form 362 with up to 11 days per page and punched cards with these data for use in time series analysis.

Par. 224-230, 232-234, and 240-245; procedures for calculating lunitidal intervals, mean heights, and ranges are handled routinely in the automated procedures described above. Par. 233 includes instructions to check the higher high and lower low of each solar day, adding a check or omitting it on the days of one tide per solar day in accordance with sequence on either side (day before and after). This led invariably to either 29 or 31 higher highs in a 60-tide month, an expedient but unsatisfactory result of tabulating a lunar phenomenon in solar time. It would have been difficult to program this method; furthermore, it was not good science. It was easier and more satisfactory to pair the high waters in sets of two (two per lunar day) and to select the higher of each pair for inclusion as higher high. A comparable treatment is applied to the low waters.

(c) Manual of Harmonic Constant Reductions

C&GS Special Publication No. 260, 1952 (Reprinted 1976)

This manual provides a mechanism for computing non-harmonic tidal quantities (lunitidal intervals, ranges, and water levels) from tidal harmonic constants (amplitudes and phases). In the past, these calculations have been hand-calculated on form 180, using the harmonic constants as input and the tables in S.P. 260.

The same calculations are now done routinely in a computer program. Those tables that are based on formulas are not used in the computer process; those derived empirically are still used: the program includes table look-up and interpolation.

(d) Tidal Datum Planes

C&GS Special Publication No. 135, Revised (1951) Edition

H. A. Marmer

Although the content of this publication is essentially unchanged, more recent studies of sea level trends have used computers in applying low pass filters (rather than moving means) to smooth the large year-to-year variations in computing secular trends and in calculating the standard error of the slope and the standard error of estimate (standard deviation from line of regression). Examples of such computer applications are found in Hicks and Crosby (1974).

Although it is not mentioned in the above manual, outstanding research in correlating sea level with meteorological factors was done in the Coast and Geodetic Survey over a century ago (Ferrel, 1871). Modern research along these lines, for example Hamon, Godfrey, and Greig (1975) depends heavily on computers for regression studies and for numerical modeling.

3. OTHER APPLICATIONS

As a means of monitoring the accuracy of tide predictions, a computer program is used to compare observed and predicted times and heights of high and low waters; an international format is used for the input series. The program provides statistics of the differences for times, heights, ranges, etc.

Various time series analysis programs are used for research purposes. These include manipulation of observed and predicted series to obtain time series of residuals (observed minus predicted) and harmonic and spectral analysis of the residual series.

Programs for response analysis and prediction, as used by Munk and Cartwright (1966), are available in National Ocean Survey, but these are not used routinely at this time.

4. EXTENDED S.P. 98, TABLES NOs. 4, 14, AND 15; YEARS 2000 THROUGH 2025.

year	S	P	H	P(1)	N	year
2000	211.74	83.29	279.97	282.94	125.07	2000
2001	354.30	124.06	280.72	282.96	105.69	2001
2002	123.69	164.73	280.48	282.97	86.36	2002
2003	253.07	205.39	280.24	282.99	67.03	2003
2004	22.46	246.05	280.00	283.01	47.70	2004
2005	165.02	286.83	280.75	283.03	28.32	2005
2006	294.40	327.49	280.51	283.04	8.99	2006
2007	63.79	8.15	280.27	283.06	349.67	2007
2008	193.17	48.81	280.03	283.08	330.34	2008
2009	335.73	89.59	280.78	283.09	310.96	2009
2010	105.12	130.25	280.54	283.11	291.63	2010
2011	234.50	170.91	280.30	283.13	272.30	2011
2012	3.89	211.57	280.06	283.15	252.97	2012
2013	146.45	252.35	280.81	283.16	233.59	2013
2014	275.83	293.01	280.57	283.18	214.26	2014
2015	45.22	333.67	280.33	283.20	194.94	2015
2016	174.60	14.34	280.09	283.22	175.61	2016
2017	317.16	55.11	280.84	283.23	156.23	2017
2018	86.55	95.77	280.60	283.25	136.90	2018
2019	215.93	136.43	280.36	283.27	117.57	2019
2020	345.32	177.10	280.12	283.28	98.24	2020
2021	127.88	217.87	280.87	283.30	78.86	2021
2022	257.26	258.53	280.63	283.32	59.53	2022
2023	26.65	299.20	280.39	283.34	40.20	2023
2024	156.03	339.86	280.15	283.35	20.88	2024
2025	298.60	20.63	280.90	283.37	1.49	2025

TABLE 14 (SP-98)

	MONTH	1	1	1	1	1	1	1	1	1	1	1	1
	DAY	1	1	1	1	1	1	1	1	1	1	1	1
	YEAR	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011
1	J(1)	.9532	1.0119	1.0654	1.1090	1.1407	1.1593	1.1651	1.1585	1.1387	1.1064	1.0620	1.0079
2	K(1)	.9626	1.0023	1.0397	1.0709	1.0942	1.1081	1.1125	1.1075	1.0928	1.0690	1.0373	.9996
3	K(2)	.8930	.9839	1.0822	1.1748	1.2507	1.2993	1.3151	1.2971	1.2459	1.1689	1.0754	.9773
4	L(2)	1.2400	1.0586	.7052	.9428	1.3094	1.1552	.6399	.8668	1.2907	1.1867	.7790	.8483
5	M(1)	.8804	1.4838	2.0515	1.8354	1.1277	1.5660	2.2503	2.0169	1.1809	1.3818	1.9712	1.8159
6	M(2)	1.0162	1.0041	.9916	.9802	.9710	.9652	.9633	.9654	.9716	.9809	.9925	1.0050
7	M(3)	1.0244	1.0062	.9874	.9704	.9568	.9482	.9454	.9486	.9576	.9715	.9887	1.0075
8	M(4)	1.0327	1.0082	.9833	.9607	.9428	.9315	.9279	.9320	.9439	.9621	.9850	1.0100
9	M(6)	1.0494	1.0124	.9750	.9417	.9154	.8991	.8938	.8998	.9171	.9438	.9775	1.0150
10	M(8)	1.0664	1.0165	.9668	.9230	.8888	.8677	.8610	.8686	.8910	.9257	.9701	1.0201
11	N(2)	1.0162	1.0041	.9916	.9802	.9710	.9652	.9633	.9654	.9716	.9809	.9925	1.0050
12	2N(2)	1.0162	1.0041	.9916	.9802	.9710	.9652	.9633	.9654	.9716	.9809	.9925	1.0050
13	O(1)	.9393	1.0037	1.0641	1.1147	1.1527	1.1755	1.1827	1.1746	1.1503	1.1116	1.0602	.9993
14	OO(1)	.8026	1.0029	1.2253	1.4418	1.6245	1.7439	1.7831	1.7386	1.6127	1.4278	1.2098	.9880
15	P(1)	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
16	Q(1)	.9393	1.0037	1.0641	1.1147	1.1527	1.1755	1.1827	1.1746	1.1503	1.1116	1.0602	.9993
17	2Q(1)	.9393	1.0037	1.0641	1.1147	1.1527	1.1755	1.1827	1.1746	1.1503	1.1116	1.0602	.9993
18	R(2)	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
19	S(1)	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
20	S(2)	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
21	S(4)	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
22	S(6)	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
23	T(2)	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
24	LAMBDA(2)	1.0162	1.0041	.9916	.9802	.9710	.9652	.9633	.9654	.9716	.9809	.9925	1.0050
25	MU(2)	1.0162	1.0041	.9916	.9802	.9710	.9652	.9633	.9654	.9716	.9809	.9925	1.0050
26	NU(2)	1.0162	1.0041	.9916	.9802	.9710	.9652	.9633	.9654	.9716	.9809	.9925	1.0050
27	RHO(1)	.9393	1.0037	1.0641	1.1147	1.1527	1.1755	1.1827	1.1746	1.1503	1.1116	1.0602	.9993
28	MK(3)	.9782	1.0064	1.0310	1.0497	1.0624	1.0695	1.0716	1.0692	1.0617	1.0486	1.0295	1.0046
29	2MK(3)	.9941	1.0106	1.0223	1.0289	1.0316	1.0322	1.0323	1.0322	1.0315	1.0286	1.0217	1.0096
30	MN(4)	1.0327	1.0082	.9833	.9607	.9428	.9315	.9279	.9320	.9439	.9621	.9850	1.0100
31	MS(4)	1.0162	1.0041	.9916	.9802	.9710	.9652	.9633	.9654	.9716	.9809	.9925	1.0050
32	2SM(2)	1.0162	1.0041	.9916	.9802	.9710	.9652	.9633	.9654	.9716	.9809	.9925	1.0050
33	MF	.8683	1.0033	1.1419	1.2678	1.3684	1.4318	1.4522	1.4290	1.3620	1.2598	1.1325	.9936
34	MSF	1.0162	1.0041	.9916	.9802	.9710	.9652	.9633	.9654	.9716	.9809	.9925	1.0050
35	MM	1.0548	1.0124	.9690	.9295	.8979	.8780	.8716	.8789	.8999	.9320	.9719	1.0155
36	SA	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
37	SSA	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

TABLE 14 (SP-98)

		1	1	1	1	1	1	1	1	1	1	1	1
	MONTH	1	1	1	1	1	1	1	1	1	1	1	1
	DAY	1	1	1	1	1	1	1	1	1	1	1	1
	YEAR	2012	2013	2014	2015	2016	2017	2018	2019	2020	2021	2022	2023
1	J(1)	.9490	.8929	.8492	8276	.8342	.8667	.9174	.9761	1.0335	1.0835	1.1228	1.1494
2	K(1)	.9598	.9232	.8954	.8820	.8861	.9064	.9390	.9780	1.0173	1.0526	1.0810	1.1007
3	K(2)	.8871	.8153	.7676	.7465	.7528	.7858	.8451	.9266	1.0216	1.1191	1.2069	1.2731
4	L(2)	1.1644	1.2046	1.0216	.8783	9704	1 1653	1.2038	9582	.7341	1.0647	1.3145	1.0392
5	M(1)	1.1234	.8780	1.3201	1.5572	1.4049	.9652	.9341	1.6117	1.9813	1.5937	1.0586	1.7587
6	M(2)	1.0170	1.0272	1 0343	1.0377	1.0367	1.0315	1.0229	1.0117	.9993	.9870	.9763	.9683
7	M(3)	1.0256	1.0411	1.0519	1.0570	1.0555	1.0477	1.0345	1.0175	.9989	.9806	.9646	.9528
8	M(4)	1.0343	1.0551	1.0699	1.0767	1.0747	1.0641	1.0463	1.0235	.9985	.9742	.9531	.9376
9	M(6)	1.0520	1.0838	1.1066	1.1173	1.1141	1.0977	1.0702	1.0354	.9978	.9615	.9305	.9078
10	M(8)	1.0699	1.1132	1.1446	1.1593	1.1549	1.1323	1.0947	1.0475	.9970	.9491	.9084	.8791
11	N(2)	1.0170	1.0272	1.0343	1.0377	1.0367	1.0315	1.0229	1.0117	.9993	.9870	.9763	.9683
12	2N(2)	1.0170	1.0272	1.0343	1.0377	1.0367	1.0315	1.0229	1.0117	.9993	.9870	.9763	.9683
13	O(1)	.9348	.8749	.8290	.8066	.8134	.8473	.9010	.9643	1.0278	1.0849	1.1311	1.1633
14	OO(1)	.7897	.6347	.5325	.4873	.5008	.5716	.6990	.8761	1.0874	1.3108	1.5185	1.6792
15	P(1)	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
16	Q(1)	.9348	.8749	.8290	.8066	.8134	.8473	.9010	.9643	1.0278	1.0849	1.1311	1.1633
17	2Q(1)	.9348	.8749	.8290	.8066	.8134	.8473	9010	.9643	1.0278	1.0849	1.1311	1.1633
18	R(2)	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
19	S(1)	1.0000	1.0000	1.0000	1.0000	1 0000	1.0000	1 0000	1.0000	1.0000	1.0000	1.0000	1.0000
20	S(2)	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
21	S(4)	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
22	S(6)	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
23	T(2)	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1 0000	1.0000	1.0000	1.0000	1.0000
24	LAMBDA(2)	1.0170	1.0272	1.0343	1.0377	1.0367	1.0315	1 0229	1.0117	.9993	.9870	.9763	.9683
25	MU(2)	1.0170	1.0272	1.0343	1.0377	1.0367	1.0315	1.0229	1.0117	.9993	.9870	.9763	.9683
26	NU(2)	1.0170	1.0272	1.0343	1.0377	1.0367	1.0315	1.0229	1.0117	.9993	.9870	.9763	.9683
27	RHO(1)	.9348	.8749	.8290	.8066	.8134	.8473	.9010	.9643	1.0278	1.0849	1.1311	1.1633
28	MK(3)	.9761	.9483	.9262	.9152	.9186	.9350	.9605	.9894	1.0165	1.0389	1.0553	1.0658
29	2MK(3)	.9928	.9740	.9580	.9497	.9522	.9645	.9825	1.0009	1.0157	1.0254	1.0303	1.0320
30	MN(4)	1.0343	1.0551	1.0699	1.0767	1.0747	1.0641	1.0463	1.0235	.9985	.9742	.9531	.9376
31	MS(4)	1.0170	1.0272	1.0343	1.0377	1.0367	1.0315	1.0229	1.0117	.9993	.9870	.9763	.9683
32	2SM(2)	1.0170	1.0272	1.0343	1.0377	1.0367	1.0315	1.0229	1.0117	.9993	.9870	.9763	.9683
33	MF	.8592	.7451	.6644	.6269	.6382	.6960	.7936	.9191	1.0572	1.1925	1.3106	1.3977
34	MSF	1.0170	1.0272	1.0343	1.0377	1.0367	1.0315	1.0229	1.0117	.9993	.9870	.9763	.9683
35	MM	1.0577	1.0934	1.1188	1.1305	1.1270	1.1089	1.0782	1.0389	.9955	.9531	.9160	.8887
36	SA	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
37	SSA	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

TABLE 14 (SP-98)

	MONTH	1	1
	DAY	1	1
	YEAR	2024	2025
1	J(1)	1.1630	1.1641
2	K(1)	1.1109	1.1117
3	K(2)	1.3093	1 3122
4	L(2)	.5919	1.0006
5	M(1)	2.2834	1.8472
6	M(2)	.9640	.9636
7	M(3)	.9464	.9459
8	M(4)	.9292	.9286
9	M(6)	.8957	.8948
10	M(8)	.8634	.8622
11	N(2)	.9640	.9636
12	2N(2)	.9640	.9636
13	O(1)	1.1801	1.1814
14	OO(1)	1.7688	1.7759
15	P(1)	1.0000	1.0000
16	Q(1)	1.1801	1.1814
17	2Q(1)	1.1801	1.1814
18	R(2)	1.0000	1.0000
19	S(1)	1.0000	1.0000
20	S(2)	1.0000	1.0000
21	S(4)	1.0000	1.0000
22	S(6)	1.0000	1.0000
23	T(2)	1.0000	1.0000
24	LAMBDA(2)	.9640	.9636
25	MU(2)	.9640	.9636
26	NU(2)	.9640	.9636
27	RHO(1)	1.1801	1.1814
28	MK(3)	1.0709	1.0713
29	2MK(3)	1.0323	1.0323
30	MN(4)	.9292	.9286
31	MS(4)	.9640	.9636
32	2SM(2)	.9640	.9636
33	MF	1.4448	1.4485
34	MSF	.9640	.9636
35	MM	.8739	.8728
36	SA	1.0000	1.0000
37	SSA	1.0000	1.0000

TABLE 15 (SP-98)

		2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011
	MONTH	1	1	1	1	1	1	1	1	1	1	1	1
	DAY	1	1	1	1	1	1	1	1	1	1	1	1
	YEAR	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011
1	J(1)	125.77	228.00	317.57	48.19	139.58	245.48	337.53	69.60	161.42	266.80	357.38	86.86
2	K(1)	1.45	1.83	2.20	3.38	5.14	8.28	10.59	12.90	15.02	17.74	18.87	19.19
3	K(2)	183.35	183.60	183.98	186.17	189.77	196.26	201.19	206.13	210.59	216.07	218.17	218.40
4	L(2)	83.71	275.07	95.26	260.72	105.01	306.24	133.06	286.93	131.87	331.47	160.87	324.68
5	M(1)	251.38	167.55	62.95	316.21	237.35	169.81	70.22	323.53	239.48	174.74	77.28	328.54
6	M(2)	134.54	210.72	311.50	52.52	153.76	230.78	332.26	73.70	175.08	251.92	352.94	93.72
7	M(3)	21.81	316.08	287.25	258.78	230.64	166.17	138.39	110.55	82.62	17.88	349.41	320.58
8	M(4)	269.08	61.44	263.00	105.04	307.52	101.56	304.52	147.40	350.16	143.84	345.88	187.44
9	M(6)	43.62	272.16	214.50	157.56	101.28	332.34	276.78	221.10	165.24	35.76	338.82	281.16
10	M(8)	178.16	122.88	166.00	210.08	255.04	203.12	249.04	294.80	340.32	287.68	331.76	14.88
11	N(2)	6.09	340.48	352.54	4.84	17.35	352.59	5.35	18.06	30.72	5.78	18.07	30.13
12	2N(2)	237.64	110.24	33.58	317.16	240.94	114.40	38.44	322.42	246.36	119.64	43.20	326.54
13	O(1)	137.22	212.96	312.89	52.01	150.59	223.49	321.64	59.74	158.02	231.26	330.43	70.45
14	OO(1)	37.42	322.56	224.33	129.01	35.75	331.09	239.60	148.18	56.10	350.06	254.59	156.09
15	P(1)	350.03	349.28	349.52	349.76	350.00	349.25	349.49	349.73	349.97	349.22	349.46	349.70
16	Q(1)	8.77	342.72	353.93	4.33	14.18	345.30	354.73	4.10	13.66	345.12	355.56	6.86
17	2Q(1)	240.32	112.48	34.97	316.65	237.77	107.11	27.82	308.46	229.30	98.98	20.69	303.27
18	R(2)	177.03	177.76	177.51	177.25	176.99	177.72	177.47	177.21	176.95	177.69	177.43	177.17
19	S(1)	180.00	180.00	180.00	180.00	180.00	180.00	180.00	180.00	180.00	180.00	180.00	180.00
20	S(2)	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00
21	S(4)	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00
22	S(6)	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00
23	T(2)	2.97	2.24	2.49	2.75	3.01	2.28	2.53	2.79	3.05	2.31	2.57	2.83
24	LAMBDA(2)	49.63	307.64	218.96	130.50	42.27	301.13	213.13	125.10	37.00	295.68	207.23	118.53
25	MU(2)	271.00	63.56	265.08	106.86	308.84	102.24	304.48	146.66	348.80	142.02	343.78	185.32
26	NU(2)	39.45	293.80	224.04	154.54	85.25	340.43	271.39	202.30	133.16	28.16	318.65	248.91
27	RHO(1)	42.13	296.04	225.43	154.03	82.08	333.14	260.77	188.34	116.10	7.50	296.14	225.64
28	MK(3)	135.99	212.55	313.70	55.90	158.90	239.06	342.85	86.60	190.10	269.66	11.81	112.91
29	2MK(3)	267.63	59.61	260.80	101.66	302.38	93.28	293.93	134.50	335.14	126.10	327.01	168.25
30	MN(4)	140.63	191.20	304.04	57.36	171.11	223.37	337.61	91.76	205.80	257.70	11.01	123.85
31	MS(4)	134.54	210.72	311.50	52.52	153.76	230.78	332.26	73.70	175.08	251.92	352.94	93.72
32	2SM(2)	225.46	149.28	48.50	307.48	206.24	129.22	27.74	286.30	184.92	108.08	7.06	266.28
33	MF	40.10	324.80	225.72	128.50	32.58	323.80	228.98	134.22	39.04	329.40	232.08	132.82
34	MSF	225.46	149.28	48.50	307.48	206.24	129.22	27.74	286.30	184.92	108.08	7.06	266.28
35	MM	128.45	230.24	318.96	47.68	136.41	238.19	326.91	55.64	144.36	246.14	334.87	63.59
36	SA	279.97	280.72	280.48	280.24	280.00	280.75	280.51	280.27	280.03	280.78	280.54	280.30
37	SSA	199.94	201.44	200.96	200.48	200.00	201.50	201.02	200.54	200.06	201.56	201.08	200.60

TABLE 15 (SP-98)

	MONTH	1	1	1	1	1	1	1	1	1	1	1	1
	DAY	1	1	1	1	1	1	1	1	1	1	1	1
	YEAR	2012	2013	2014	2015	2016	2017	2018	2019	2020	2021	2022	2023
1	J(1)	174.93	275.35	360.00	83.36	166.40	264.31	349.73	76.90	165.65	269.73	.68	92.27
2	K(1)	18.49	17.68	14.84	11.27	7.55	5.25	2.86	1.57	1.35	3.06	4.49	6.42
3	K(2)	216.52	214.56	208.96	202.37	195.59	191.34	186.41	183.38	182.49	185.60	188.39	192.39
4	L(2)	157.82	348.12	182.73	3.67	182.09	1.98	202.15	39.81	211.35	17.28	224.34	74.17
5	M(1)	231.31	159.53	72.76	322.97	211.44	102.42	42.71	314.60	206.57	90.26	24.62	317.01
6	M(2)	194.24	270.18	10.36	110.42	210.46	286.20	26.40	126.82	227.46	303.98	45.10	146.40
7	M(3)	291.36	225.27	195.54	165.63	135.69	69.30	39.60	10.23	341.19	275.97	247.65	219.60
8	M(4)	28.48	180.36	20.72	220.84	60.92	212.40	52.80	253.64	94.92	247.96	90.20	292.80
9	M(6)	222.72	90.54	31.08	331.26	271.38	138.60	79.20	20.46	322.38	191.94	135.30	79.20
10	M(8)	56.96	.72	41.44	81.68	121.84	64.80	105.60	147.28	189.84	135.92	180.40	225.60
11	N(2)	41.92	16.08	27.54	38.87	50.20	24.15	35.62	47.32	59.24	33.97	46.37	58.95
12	2N(2)	249.60	121.98	44.72	327.32	249.94	122.10	44.84	327.82	251.02	123.96	47.64	331.50
13	O(1)	171.63	248.93	353.18	98.61	204.32	283.94	27.45	129.42	230.03	304.26	43.15	141.58
14	OO(1)	53.59	333.57	221.18	105.01	347.96	260.58	150.45	45.38	304.83	235.18	140.75	48.06
15	P(1)	349.94	349.19	349.43	349.67	349.91	349.16	349.40	349.64	349.88	349.13	349.37	349.61
16	Q(1)	19.31	354.83	10.36	27.06	44.06	21.89	36.67	49.92	61.81	34.25	44.42	54.13
17	2Q(1)	226.99	100.73	27.54	315.51	243.80	119.84	45.89	330.42	253.59	124.24	45.69	326.68
18	R(2)	176.91	177.65	177.39	177.13	176.87	177.61	177.35	177.09	176.84	177.57	177.31	177.05
19	S(1)	180.00	180.00	180.00	180.00	180.00	180.00	180.00	180.00	180.00	180.00	180.00	180.00
20	S(2)	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00
21	S(4)	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00
22	S(6)	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00
23	T(2)	3.09	2.35	2.61	2.87	3.13	2.39	2.65	2.91	3.16	2.43	2.69	2.95
24	LAMBDA(2)	29.58	287.36	198.06	108.65	19.22	276.79	187.52	98.46	9.64	267.99	179.63	91.47
25	MU(2)	26.58	178.90	19.84	220.64	61.44	213.56	54.50	255.68	97.06	249.96	91.84	293.88
26	NU(2)	178.90	73.00	2.66	292.19	221.70	115.61	45.28	335.18	265.28	159.97	90.57	21.33
27	RHO(1)	156.29	51.75	345.48	280.38	215.56	113.35	46.33	337.78	267.85	160.25	88.62	16.51
28	MK(3)	212.73	287.86	25.20	121.69	218.01	291.45	29.26	128.39	228.81	307.04	49.59	152.82
29	2MK(3)	9.99	162.68	5.88	209.57	53.37	207.15	49.94	252.07	93.57	244.90	85.71	286.38
30	MN(4)	236.16	286.26	37.90	149.29	260.66	310.35	62.02	174.14	286.70	337.95	91.47	205.35
31	MS(4)	194.24	270.18	10.36	110.42	210.46	286.20	26.40	126.82	227.46	303.98	45.10	146.40
32	2SM(2)	165.76	89.82	349.64	249.58	149.54	73.80	333.60	233.18	132.54	56.02	314.90	213.60
33	MF	30.98	312.32	204.00	93.20	341.82	258.32	151.50	47.98	307.40	235.46	138.80	43.24
34	MSF	165.76	89.82	349.64	249.58	149.54	73.80	333.60	233.18	132.54	56.02	314.90	213.60
35	MM	152.32	254.10	342.82	71.55	160.26	262.05	350.78	79.50	168.22	270.01	358.73	87.45
36	SA	280.06	280.81	280.57	280.33	280.09	280.84	280.60	280.36	280.12	280.87	280.63	280.39
37	SSA	200.12	201.62	201.14	200.66	200.18	201.68	201.20	200.72	200.24	201.74	201.26	200.78

TABLE 15 (SP-98)

	MONTH	1	1
	DAY	1	1
	YEAR	2024	2025
1	J(1)	184.23	290.38
2	K(1)	8.66	11.98
3	K(2)	197.13	204.10
4	L(2)	246.04	41.02
5	M(1)	213.12	96.20
6	M(2)	247.82	324.90
7	M(3)	191.73	127.35
8	M(4)	135.64	289.80
9	M(6)	23.46	254.70
10	M(8)	271.28	219.60
11	N(2)	71.65	46.93
12	2N(2)	255.48	128.96
13	O(1)	239.76	312.49
14	OO(1)	316.36	252.33
15	P(1)	349.85	349.10
16	Q(1)	63.59	34.52
17	2Q(1)	247.42	116.55
18	R(2)	176.80	177.53
19	S(1)	180.00	180.00
20	S(2)	.00	.00
21	S(4)	.00	.00
22	S(6)	.00	.00
23	T(2)	3.20	2.47
24	LAMBDA(2)	3.41	262.33
25	MU(2)	136.06	289.50
26	NU(2)	312.23	207.47
27	RHO(1)	304.17	195.06
28	MK(3)	256.48	336.88
29	2MK(3)	126.98	277.82
30	MN(4)	319.47	11.83
31	MS(4)	247.82	324.90
32	2SM(2)	112.18	35.10
33	MF	308.30	239.92
34	MSF	112.18	35.10
35	MM	176.17	277.97
36	SA	280.15	280.90
37	SSA	200.30	201.80

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6. APPENDIX A - COMPUTER PROGRAMS

A. Tide predictions

B. Tidal analysis by least squares

A. Tide Predictions

A FORTRAN Program for the Calculation of Hourly Values of Astronomical Tide and Time and Height of High and Low Water

The height of the tide at any time, as given by the Manual of Harmonic Analysis and Prediction of Tides, Special Publication No. 98, U.S. Department of Commerce may be written:

$$h = H_o + \sum_{n}^{N} f_n H_n \cos [a_n t + (V_o + u)_n - K_n'] \qquad \text{(equation 1)}$$

where

h = height of tide at any time t.

H_o = mean height of water level above datum used for prediction.

H_n = mean amplitude of any constituent A_n.

f_n = factor for reducing mean amplitude to year of prediction.

a_n = hourly speed of constituent A_n.

t = time, in hours, reckoned from beginning of year of prediction.

$(V_o+u)_n$ = Greenwich equilibrium argument of constituent A_n when t = 0.

K'_n = modified epoch of constituent A_n.

N = number of constituents used for the particular station.

A listing of the source statements for a program for computing hourly values and times and heights of high and low astronomical tide is presented. The program is written in FORTRAN language. It should be adaptable to any computer with adequate memory and which utilizes the FORTRAN system.

The steps in the program are described following the listing. The steps are keyed to the source statements by the numbers added to the right edge of the listing.

```
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C      MARINE PREDICTIONS BRANCH, OCEANOGRAPHIC DIVISION, NOS
C      PROGRAM PREDICT.NTP4-SHORT
C      THIS PROGRAM PREDICTS HOURLY TIDAL HEIGHTS AND/OR DAILY
C      TIMES AND HEIGHTS OF TIDAL EXTREMES.
C      TIMES ARE CALCULATED TO THE NEAREST MINUTE AND HEIGHTS
C      ARE CALCULATED TO THE NEAREST TENTH OF A FOOT.
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
       DIMENSION A(37),AMP(37),EPOC(37),XODE(37),VPU(37),MO(13),
     1 SP(37),NBDAY(13),XCOS(2049),SPD(37),ARG(37),NEDAY(13),S(37),
     2 TABHR(25),ANG(37),KDAY(32),STORX(816),EXTIM(600),JXTIM(300),
     3 XHT(300),EPOCH(37),AMPA(37),XXHT(18),JJXTI(18),IYR(5),NUM(5),
     4 ISTA(6),NO(6),NHT(300)
       DATA A/      28.9841042, 30.0000000, 28.4397295, 15.0410686,           1
     1 57.9682084, 13.9430356, 86.9523127, 44.0251729, 60.0000000,
     2 57.4238337, 28.5125831, 90.0000000, 27.9682084, 27.8953548,
     3 16.1391017, 29.4556253, 15.0000000, 14.4966939, 15.5854433,
     4   .5443747,   .0821373,   .0410686,  1.0158958,  1.0980331,
     5 13.4715145, 13.3986609, 29.9589333, 30.0410667, 12.8542862,
     6 14.9589314, 31.0158958, 43.4761563, 29.5284789, 42.9271398,
     7 30.0821373,115.9364169, 58.9841042/
       DATA (TABHR(I), I=1,24)/  -24.,   720., 1392., 2136., 2856., 3600.,    2
     1    4320., 5064., 5808., 6528., 7272., 7992.,  -24.,  720., 1416.,
     2    2160., 2880., 3624., 4344., 5088., 5832., 6552., 7296., 8016./
C      DEVELOP COSINE TABLE
       H = 0.00076699039394
       DO 1 I=1,2049                                                          3
     1 XCOS(I) = COS(H * FLOAT(I - 1))
       MS , MY , MD = 1                                                       4
       CON = 2048. / 90.
       DO 90 J = 1,37                                                         5
       A(J) = A(J) * CON
    90 CONTINUE
   100 IF (MS) 120,120,110                                                    6
   110 READ 550                                                               7
       READ 532, DATUM,IND
       READ 531, ISTA(1),NO(1),(AMP(J),EPOC(J), J=1,7),ISTA(2),NO(2),
     1 (AMP(J),EPOC(J),J=8,14),ISTA(3),NO(3),(AMP(J),EPOC(J),J=15,21),
     2 ISTA(4),NO(4),(AMP(J),EPOC(J),J=22,28),ISTA(5),NO(5),(AMP(J),
     3 EPOC(J),J=29,35),ISTA(6),NO(6),(AMP(J),EPOC(J),J=36,37)
   112 DO 113 L = 1,5
       IF (ISTA(L).NE.ISTA(L+1)) GO TO 451
   113 CONTINUE
       ISTA1 = ISTA(1)
       DO 114 L = 1,6
       IF (NO(L).NE.L) GO TO 450
   114 CONTINUE
   120 IF (MY) 131,131,125                                                    8
   125 READ 533, IYR(1),LY1,NUM(1),(XODE(J),VPU(J),J=1,8),IYR(2),LY2,         9
     1 NUM(2),(XODE(J),VPU(J),J=9,16),IYR(3),LY3,NUM(3),(XODE(J),VPU(J),
     2 J=17,24),IYR(4),LY4,NUM(4),(XODE(J),VPU(J),J=25,32),IYR(5),LY5,
```

```
     3 NUM(5),(XODE(J),VPU(J),J=33,37)
      DO 127 L = 1,4
      IF (IYR(L).NE.IYR(L+1)) GO TO 452
  127 CONTINUE
      DO 130 L = 1,5
      IF (NUM(L).NE.L) GO TO 453
  130 CONTINUE
      IYRO = MOD(IYR(1),100)
      IYR1 = IYR(1)
  131 IF (MD) 160,160,140                                                      10
  140 READ 534,    (MO(J), NBDAY(J), NEDAY(J), J = 1,12)                       11
C     SET UP TABLES FOR NON-ZERO CONSTITUENTS
  160 K = 0
      DO 180 J = 1,37                                                          12
      IF (AMP(J).EQ.0.0) GO TO 180
      K = K + 1
      AMPA(K) = AMP(J) * XODE(J)
      TEMX = VPU(J) - EPOC(J)
      IF (TEMX .GE. 0.) GO TO 171
      TEMX = TEMX + 360.
  171 EPOCH(K) = TEMX * CON
      SPD(K) = A(J)
      SP(K) = SPD(K) / 10.
      S(K) = SPD(K) / 60.
  180 CONTINUE
      NOCON = K                                                                13
C     OPERATING TABLES NOW STORED AS AMPA(K),EPOCH(K),SPD(K),SP(K),S(K)
      DO 2000 JP = 1,12
      MO(13) = MO(JP)                                                          14
      NBDAY(13) = NBDAY(JP)
      NEDAY(13) = NEDAY(JP)
      NNEDA = NEDAY(13) + 1
      IF (MO(JP)) 2005,2005,190                                                15
  190 NODAYS = NEDAY(13) - NBDAY(13) + 3                                       16
      NOD = NODAYS - 2
      NOHRS = NODAYS * 24
      HRS = NOHRS
C     DETERMINE FIRST HOUR OF TIME PERIOD
      IF (LY1) 200,200,210                                                     17
  200 K = MO(13)
      GO TO 215
  210 K =  MO(13) + 12
  215 BDAY = NBDAY(13)
      FIRST = TABHR(K) + BDAY * 24.
      NFIRST = FIRST
      DO 220 J = 1,816                                                         18
      STORX(J) = 0.
  220 CONTINUE
C     TIDE = DATUM + AMPA(K) * COS(A(K) * T + EPOCH(K))
      KOUNT , KT = 0                                                           19
      DO 380  K = 1,NOHRS
```

```
      IF (KOUNT.GT.0) GO TO 260
      KOUNT = 1
  231 DO 250 J = 1,NOCON                                              20
      ARGU = SPD(J) * FIRST + EPOCH(J)
      ARG(J) = AMOD(ARGU,8192.)
  250 CONTINUE
      GO TO 290
  260 DO 280  J = 1,NOCON                                             21
      ARG(J) = ARG(J) + SPD(J)
  270 IF (ARG(J).LT.8192.) GO TO 280
      ARG(J) = ARG(J) - 8192.
      GO TO 270
  280 CONTINUE
  290 DO 374 J = 1,NOCON                                              22
      IF (ARG(J) - 2048.) 320,320,300
  300 IF (ARG(J) - 4096.) 350,350,310
  310 IF (ARG(J) - 6144.) 360,360,330
  320 NP = ARG(J) + 1.5                                               23
      GO TO 340
  330 NP = 8192. - ARG(J) + 1.5
  340 STORX(K) = STORX(K) + AMPA(J) * XCOS(NP)                        24
      GO TO 374                                                       25
  350 NP = 4096. - ARG(J)
      GO TO 370
  360 NP = ARG(J) - 4096. + 1.5
  370 STORX(K) = STORX(K) - AMPA(J) * XCOS(NP)                        26
  374 CONTINUE                                                        27
      STORX(K) = STORX(K) + DATUM
      IF (K.NE.NOHRS) GO TO 380                                       28
      IF (KT.EQ.1) GO TO 378
      FIRST = FIRST + HRS - 1.
      KT = 1
      CHECK = STORX(K)                                                29
      STORX(K) = 0.
      GO TO 231                                                       30
  378 CKSUM = CHECK - STORX(K)                                        31
  380 CONTINUE
      GO TO (419,401,401),IND                                         32
  401 KDAY(1) = NBDAY(13)                                             33
      DO 410 I = 2,NOD
      KDAY(I) = KDAY(I-1) + 1
  410 CONTINUE
      PRINT 550                                                       34
      PRINT 556, IYR1,MO(13),DATUM,CKSUM
      PRINT 557                                                       35
      PRINT 537,    (KDAY(I), STORX(24*I-23), STORX(24*I-22), STORX(24*I
     1 -21),STORX(24*I-20), STORX(24*I-19), STORX(24*I-18), STORX(24*I-
     2 17), STORX(24*I-16), STORX(24*I-15), STORX(24*I-14), STORX(24*I-
     3 13), STORX(24*I-12), KDAY(I), STORX(24*I-11), STORX(24*I-10),
     4 STORX(24*I-9), STORX(24*I-8), STORX(24*I-7), STORX(24*I-6),
     5 STORX(24*I-5), STORX(24*I-4), STORX(24*I-3), STORX(24*I-2),
```

```
     6 STORX(24*I-1), STORX(24*I), I=1,NOD)
 419 IF (IND.EQ.3) GO TO 2000                                          36
C      COMPUTE TIMES AND HEIGHTS OF HIGH AND LOW WATERS
1000  ITEMS = 0                                                        37
     K , NST , IJOB = 1
      NOHRS = NOHRS - 1                                                38
      DO 1500 I = 1,NOHRS                                              39
      GO TO (1038,1275,1290,1426), IJOB                                40
1038  GO TO (1039,1270,1287),NST                                       41
1039  NWHOA = 7
      TIME = NFIRST * 10
      NARC = 1
      GO TO 1060
1040  IF(STORX(I) - STORX(I+1)) 1285,1045,1265                         42
1045  NWHOA = 1
      TIME = NFIRST * 10
      GO TO 1051
1050  TIME = (NFIRST + I - 2) * 10                                     43
1051  NARC = 1                                                         44
1060  STOXR = DATUM
      GO TO (1075,1100),NARC
1075  DO 1090 J = 1,NOCON
      ARGU = SP(J) * TIME + EPOCH(J)
      ARG(J) = AMOD(ARGU,8192.)
1090  CONTINUE
      GO TO 1120
1100  DO 1110 J = 1,NOCON
      ARG(J) = ARG(J) + SP(J)
1105  IF (ARG(J).LT.8192.) GO TO 1110
      ARG(J) = ARG(J) - 8192.
      GO TO 1105
1110  CONTINUE
1120  DO 1220 J = 1,NOCON
      IF (ARG(J) - 2048.) 1150,1150,1130
1130  IF (ARG(J) - 4096.) 1180,1180,1140
1140  IF (ARG(J) - 6144.) 1190,1190,1160
1150 NP = ARG(J) + 1.5
      GO TO 1170
1160 NP = 8192. - ARG(J) + 1.5
1170 STOXR = STOXR + AMPA(J) * XCOS(NP)
      GO TO 1220
1180 NP = 4096. - ARG(J) + 1.5
      GO TO 1200
1190 NP = ARG(J) - 4096. + 1.5
1200 STOXR = STOXR - AMPA(J) * XCOS(NP)
1220  CONTINUE
      GO TO (1250,1260,1275,1280,1290,1295,1400,1410,1412), NWHOA      45
1250  POINT1 = STOXR                                                   46
     NWHOA , NARC = 2
      GO TO 1060
1260  IF (POINT1 - STOXR) 1285,1060,1265
```

```
1265  NWHOA = 3                                          47
      NST = 2
1270  IF (STORX(I) - STORX(I+1)) 1050,1050,1499          48
1275  POINT1 = STOXR                                     49
      JHOL = 1
      NWHOA = 4
      NARC = 2
      TIME = TIME + 1.
      GO TO 1060
1280  IF (POINT1 - STOXR) 1470,1296,1300                 50
1285  NWHOA = 5                                          51
      NST = 3
1287  IF (STORX(I) - STORX(I+1)) 1499,1050,1050          52
1290  POINT1 = STOXR                                     53
     JHOL , NARC = 2
      NWHOA = 6
      TIME = TIME + 1.
      GO TO 1060
1295  IF (POINT1 - STOXR) 1300,1296,1470                 54
1296  OTIME = TIME                                       55
      TIME = TIME * 6. - 6.
      GO TO 1471
1300  TIME = TIME + 1.
      POINT1 = STOXR
      GO TO 1060
1310  IJOB = 2
     GO TO 1500
1315  IJOB = 3
     GO TO 1500
1400  POINT1 = STOXR                                     56
      NWHOA = 8
      NARC = 2
      TIME = TIME + 1.
      GO TO 1060
1410  IF (POINT1-STOXR) 1415,1411,1420
1411  NWHOA = 9
      POINT2 = STOXR
      GO TO 1060
1412  IF (POINT2-STOXR) 1413,1040,1414
1413 JHOL = 1
     GO TO 1425
1414 JHOL = 2
     GO TO 1425
1415  IF (STOXR - STORX(I+1)) 1285,1315,1315
1420  IF (STOXR - STORX(I+1)) 1310,1310,1265
1425  IJOB = 4
     GO TO 1500
1426  TIME = (TIME - 1.) * 6.                            57
      GO TO 1471
1428  TMPTM = EXTIM(K)                                   58
1430  K = K + 2
```

```
      ITEMS = ITEMS + 2
1431  GO TO (1285,1265), JHOL
1470  OTIME = TIME                                                    59
      TIME = TIME * 6. - 9.
1471  AM = 0                                                          60
      DO 1495 L = 1,7
      STOX = DATUM
      IF (L.GT.1) GO TO 1473
      DO 1472 J = 1,NOCON
      ARGU = S(J) * TIME + EPOCH(J)
      ARG(J) = AMOD(ARGU,8192.)
1472  CONTINUE
      GO TO 1476
1473  DO 1475 J = 1,NOCON
      ARG(J) = ARG(J) + S(J)
1474  IF (ARG(J).LT.8192.) GO TO 1475
      ARG(J) = ARG(J) - 8192.
      GO TO 1474
1475  CONTINUE
1476  DO 1486 J = 1,NOCON
      IF (ARG(J) - 2048.) 1479,1479,1477
1477  IF (ARG(J) - 4096.) 1482,1482,1478
1478  IF (ARG(J) - 6144.) 1483,1483,1480
1479 NP = ARG(J) + 1.5
      GO TO 1481
1480 NP = 8192. - ARG(J) + 1.5
1481 STOX = STOX + AMPA(J) * XCOS(NP)
      GO TO 1486
1482 NP = 4096. - ARG(J) + 1.5
      GO TO 1484
1483 NP = ARG(J) - 4096. + 1.5
1484 STOX = STOX - AMPA(J) * XCOS(NP)
1486  CONTINUE
      IF (L.EQ.1) SAVIT = STOX
      GO TO (1490,1487), JHOL
1487  IF (SAVIT.GE.STOX) GO TO 1495
      GO TO 1492
1490  IF (SAVIT.LE.STOX) GO TO 1495
1492  SAVIT = STOX
      AM = L - 1
1495  CONTINUE
      EXTIM(K) = TIME + AM                                            61
      EXTIM(K+1) = SAVIT
      IF (K.EQ.1) GO TO 1428
      IF (EXTIM(K) - TMPTM) 1496,1496,1428
1496  TIME = OTIME + 1.
      POINT1 = STOXR
      GO TO 1051
1499  IJOB = 1
1500  CONTINUE
     NITEMS , MN = 0
```

```
      DO 1504 K = 2,ITEMS,2                                           62
      MN = MN + 1
      IF (EXTIM(K)) 1501,1502,1502
1501  NHT(MN) = EXTIM(K) * 10. - 0.5
      GO TO 1504
1502  NHT(MN) = EXTIM(K) * 10. + 0.5
1504  CONTINUE
      KSW = 9000                                                      63
     J , L , KJ = 1
      KAYE = MN - 1
      DO 1515 K = 1,KAYE
      IF (KJ.EQ.2) GO TO 1513
      IF (NHT(K) - NHT(K+1)) 1505,1507,1505
1505  IF ((EXTIM(J) + 120.0).GT.EXTIM(J+2)) GO TO 1510
1506  EXTIM(L) = EXTIM(J)
      EXTIM(L+1) = EXTIM(J+1)
      L = L + 2
      J = J + 2
      NITEMS = NITEMS + 2
      GO TO 1514
1507  IF ((EXTIM(J) + 120.0).GT.EXTIM(J+2)) GO TO 1512
      IF (EXTIM(J+1) - EXTIM(J+3)) 1508,1506,1509
1508  EXTIM(J+1) = EXTIM(J+1) - 0.1
      NHT(K) = NHT(K) - 1
      GO TO 1506
1509  EXTIM(J+3) = EXTIM(J+3) - 0.1
      NHT(K+1) = NHT(K+1) - 1
      GO TO 1506
1510  IF (ABS(EXTIM(J+1) - EXTIM(J+3)).LE.0.05) GO TO 1512
      GO TO 1506
1512  IF((K-KSW).EQ.1) GO TO 1506
      KJ = 2
     GO TO 1515
1513  KSW = K
      J = J + 4
1514  KJ = 1
1515  CONTINUE
      KAY = NITEMS / 2                                                64
      J = 0
      DO 1650 K = 1,NITEMS,2
      J = J + 1
      JHRS = EXTIM(K)
      XHT(J) = EXTIM(K+1)
      JDAY = MOD(JHRS,1440)
      JHR = JDAY / 60
      JMIN = MOD(JDAY,60)
      JXTIM(J) = JHR * 100 + JMIN
      IF (XHT(J)) 1600,1650,1650                                      65
1600  IF (XHT(J).LE.-0.05) GO TO 1650
      XHT(J) = XHT(J) * (-1.0)
1650  CONTINUE
```

```
      KK = KAY                                                    66
      IF(JP.EQ.1) GO TO 1670
      IF(JXTIM(1) - NSAV) 1652,1670,1660
 1652 KA = KAY - 1
      DO 1653 KO=1,KA
      JXTIM(KO) = JXTIM(KO+1)
      XHT(KO) = XHT(KO+1)
 1653 CONTINUE
      GO TO 1670
 1660 JXTIM(KK+1)=  JXTIM(KK)
      XHT(KK+1) = XHT(KK)
      KK = KK - 1
      IF(KK.EQ.0) GO TO 1665
      GO TO 1660
 1665 JXTIM(1) = NSAV
      XHT(1) = SAV
 1670  NDAY = NBDAY(13)                                           67
       NCOUNT = 0
       NNJ = 1
 1674  PRINT 550                                                  68
       PRINT 560, IYR1,MO(13),CKSUM
       PRINT 580
 1704  DO 1750 I = 1,KAY                                          69
       IF (JXTIM(I) - JXTIM(I+1)) 1705,1710,1715
 1705  NCOUNT = NCOUNT + 1
       GO TO 1750
 1710  PRINT 581
       STOP
 1715  NLAST = NNJ + NCOUNT
 1717  PRINT 585,    NDAY,(JXTIM(J),XHT(J), J = NNJ,NLAST)
 1740  NNJ = NLAST + 1
       NCOUNT = 0
       NDAY = NDAY + 1
 1745 IF(NDAY.EQ.NNEDA) GO TO 1746
      GO TO 1750
 1746 SAV = XHT(I + 1)                                            70
      NSAV = JXTIM(I + 1)
      GO TO 2000
 1750  CONTINUE
 2000  CONTINUE
 2005  READ 538,    MS,MY,MD                                      71
 2010  IF (MS+MY+MD) 2020,2020,100
 2020  STOP
  450 PRINT 501                                                   72
      STOP
  451 PRINT 502
      STOP
  452 PRINT 503                                                   73
      STOP
  453 PRINT 504
      STOP
```

```
  501 FORMAT (27H STATION CARDS OUT OF ORDER)
  502 FORMAT (31H STATION NUMBERS NOT CONSISTENT)
  503 FORMAT (28H YEAR NUMBERS NOT CONSISTENT)
  504 FORMAT (24H YEAR CARDS OUT OF ORDER)
  530 FORMAT (7F10.7)
  531 FORMAT(2I4,F5.3,F4.1,F5.3,F4.1,F5.3,F4.1,F5.3,F4.1,F5.3,F4.1,F5.3,
     1F4.1,F5.3,F4.1)
  532 FORMAT (F6.3,I2)
  533 FORMAT (I4,2I2,F4.3,F4.1,F4.3,F4.1,F4.3,F4.1,F4.3,F4.1,F4.3,F4.1,
     1F4.3,F4.1,F4.3,F4.1,F4.3,F4.1)
  534 FORMAT (36I2)
  537 FORMAT (I9,12F9.1)
  538 FORMAT (3I4)
  550 FORMAT (72H
     1                   )
  556 FORMAT (7X,30HPREDICTED HOURLY HEIGHTS  YEAR,I5,7H  MONTH,I3,7H  D
     1ATUM,F7.2,4X,34H  *NOAA, NATIONAL OCEAN SURVEY*   ,3X,F10.7/)
  557 FORMAT (7X,110HDAY   HOURS    HOURS    HOURS    HOURS    HOURS
     1HOURS    HOURS    HOURS    HOURS    HOURS    HOURS    HOURS/14X,10
     23H0/12     1/13     2/14     3/15     4/16     5/17     6/18     7
     3/19     8/20     9/21    10/22    11/23/)
  560 FORMAT (7X,44HTIDE PREDICTIONS (HIGH AND LOW WATERS)  YEAR,I5,7H
     1MONTH,I3,36X,F10.7)
  580  FORMAT (112H       DAY      TIME      HT.      TIME      HT.      TIME
     1      HT.      TIME      HT.      TIME      HT.      TIME      HT.)
  581  FORMAT (28H CONSECUTIVE TIDES SAME TIME)
  585  FORMAT (1H0,I9,6(5X,I4,F8.1)/10X,6(5X,I4,F8.1)/10X,6(5X,I4,F8.1))
       END
```

1. Constituent speeds are stored in the array A(J). The program is compiled to run with 37 constitutents. This number can be increased by a change of the DIMENSION statement and a change of the indexing of the appropriate statements.

2. Table of starting hours for each month. TABHR(1) to TABHR(12) are for a regular year, TABHR(13) to TABHR(24) are for leap years.

3. Set up cosine table for table lookup. The interval 0 - $\pi/2$ is divided into 2,048 parts.

4. Initialize MS, MY, and MD, the control words which determine if more than one problem is to be done. Specify that constants for the station, year, and date will be read in. After the first set of calculations these variables may be set to zero if the station, year, or date are not being changed.

5. Convert the constituent speeds to units such that 2,048 units are equal to $\pi/2$.

6. Check to see if station constants are to be read.

7. Read station name from punched card (72H). Read in the datum plane (DATUM) and control word (IND). The tide heights are calculated with reference to mean sea level datum plane. The amount in feet specified by DATUM is added to the calculations before they are printed. The control word IND determines if hourly calculations, high and low water calculations, or both are to be printed. IND is set to 1 for highs and lows only, 2 for both hourly values and high and lows, and 3 for hourly values only.

The amplitude of each constituent (AMP(J)), and the modified epoch of each constituent (EPOCH(J)) are read in for the station. These require six punched cards. The first word on each card is the assigned station number. The second word is the card number, 1 through 6. These cards must be in the correct numerical order so that the amplitudes and epochs are read in for the appropriate constituents. The station numbers are compared for consistency and the order of the six cards is checked.

8. Determine if year constants are to be read in.

9. Read in the node factor (XODE(J)) and the equilibrium argument (VPU(J)) for each constituent for the particular year. This requires five cards. The first word is the year, the second specifies whether the year of calculation is a leap year or a non-leap year (1 for leap year and 0 for non-leap year). The third word is the card number, 1 through 5. The cards are checked for year consistency and for card order.

10. Determine if a date card is to be read in.

11. Read in the date control card consisting of 12 time periods. MO(J) is month, NBDAY(J) is the beginning day, and NEDAY(J) is the ending day. If calculations are for less than 12 time periods, the unused portion of the card is left blank. The unused words (MO(J), NBDAY(J), and NEDAY(J) are therefore read in as zero.

12. Examine amplitudes of the constitutents (AMP(J)) for zero and form a set of tables consisting of AMPA(K) which is $f_n H_n$ in equation 1, EPOCH(K) which

is $(V_0 + u)_n - K'_n$ in equation 1, and SPD(K) which is a_n in equation 1. The resulting set of constants is for the constituents of non-zero amplitude. Also constituent speed in tenths of hours are computed and stored as SP(K), constituent speeds in 1/60th of an hour (minutes) are computed and stored in S(K).

13. Set NOCON = K, the number of non-zero constituents, which is the number of constituents to be used in the calculations.

14. Put the month, beginning, and ending dates MO(13), NBDAY(13), and NEDAY(13).

15. Check for blank (zero) month number which indicates calculations for the last time period have been completed.

16. Determine number of days (NODAYS) and number of hours (NOHRS) for which calculations are to be made.

17. Check whether year is leap year or non-leap year. Determine the first hour number of the year for which calculations are to be made (FIRST). The first hour number of each month is saved as NFIRST.

18. Zero storage array for hourly heights.

19. Zero counters, and check to see if you want the height for the first time in the period being predicted.

20. Determine phase angle of each constituent for the first hour of calculation. These arguments are then reduced to values of less than 2π.

21. Determine for all hours after the first the phases of the constituents by adding the speed of each constituent SPD(J) to the previous phase (ARG(J)). These also are reduced to less than 2π.

22. Determine the quadrant of the phase of the constituent. This is necessary as the program utilizes a cosine table of 2,049 values, from 0 to $\pi/2$.

23. NP is the value used to enter the cosine table for a particular constituent; NP is determined by rounding to the next highest integer.

24. Accumulate the value of the tide for a particular hour, STORX(K), for those constituents which are in an increasing phase.

25. Same as 23.

26. Same as 24, except for constituents which are in a decreasing phase.

27. Add datum plane to the tide calculation.

28. Find time of last hour for which calculations are made. This is used for calculating the tide height for the last hour the second time by the method used in the first hour in step 20. This independent calculation of the last hour is compared to the value for the last hour obtained by the short cut method of step 21.

29. Store last height predicted in CHECK. This is used for step 28.

30. Transfer for calculation of the last hour tide height.

31. Find CKSUM, the difference between the two calculations of the last hour tide. Should be very near zero.

32. Check to see if the hourly heights are to be printed.

33. Store day numbers for hourly printout in KDAY.

34. Write heading lines consisting of the station name, the year, month, datum, number of constituents, and CKSUM.

35. Write out hourly calculations of tidal heights.

36. Check to determine if hourly values only are desired.

37. Initialize ITEMS to zero. ITEMS builds up to twice the number of extreme tides.

38. Set number of 1-hour time intervals.

39. Begin search for tidal extremes.

40. Test to determine if first hour is being considered.

41. Initialize for first hour.

42. Check to see if the approaching extreme is a high or a low water.

43. States that an extreme value will occur between TIME - 1 and TIME + 1 hours.

44. Compute tide heights at tenth of an hour intervals. Similar to computation of hourly tide heights.

45. Transfer, depending upon whether the program is searching for a high or low water.

46. Set flags, increment time by one tenth of an hour, jump back and predict height with new time.

47. Initialize for low water search.

48. Search for low water.

49. Same as 46.

50. Check to determine if low water has been found.

51. Initialize for search of high water.

52. Search for high water.

53. Same as 46.

54. Check to determine if high water has been found.

55. Same as 46.

56. Determine if first tide extreme is high or low water.

57. Reset time back 6 minutes and start search for minute of extreme.

58. Transfer appropriately to search for high or low water. Increment ITEMS by 2 to indicate an extreme has been found. Jump to set up counters to search for the next high or low water depending upon the value of JHOL.

59. Reset time back 3 minutes and start search for minute of extreme.

60. Determine exact minute of tidal extreme. Similar to computation of hourly heights.

61. Store time and height of tidal extreme.

62. Round value of heights to the nearest tenth of a foot.

63. Check for and eliminate consecutive tides with the same time and/or height.

64. Change times of extreme to hours and minutes.

65. Set sign of all zero heights to plus.

66. Do a monthly tie-in to make sure that extremes occurring around 0000 are not lost.

67. Set initial day number.

68. Write four heading lines.

69. Write times and heights of tidal extremes.

70. Save the time and height of the last extreme for the monthly tie-in.

71. Read MS, MY, and MD to determine if more problems are to be done. If MS, MY, and MD are all zero, all calculations are finished. If any of the three are not zero, more calculations are to follow. If MS is non-zero, new stations cards are to be loaded. If MY is non-zero, new year cards are to be loaded. If MD is non-zero, a new date control card is loaded.

72. Print out error message and stop.

73. Formats.

(b) Tidal Analysis by Least Squares

Harmonic Analysis of Tidal Data (Least Squares)

This program runs on a UNIVAC 1100/43 computer. The program generates a variable length sine table in five quadrants; and the independent variables, which are functions of time, are extracted from this table. The least squares method is employed, and the harmonic constants are derived by the use of a multiple correlation screening process which can be terminated when the regression equation contains a specified number of terms or when the next constituent will not explain a predetermined fraction of the variance. Tide heights are read into core storage from magnetic tape or cards. Other input includes constituent speeds in degrees per solar hour, node factors, and equilibrium arguments in degrees. The output is an ordered listing of harmonic constants. The order of individual values of constituent speeds, node factors, equilibrium arguments, and component (constituent) subscripts must be the same. The output is a listing of harmonic constants in the same order as the input information. The order of the 37 constituents routinely used by NOS is as follows (reading left to right):

$M_2, S_2, N_2, K_1, M_4, O_1, M_6, (MK)_3, S_4, (MN)_4,$

$Nu_2, S_6, Mu_2, (2N)_2, (OO)_1, Lambda_2, S_1, M_1,$

$J_1, Mm, Ssa, Sa, MSf, Mf, Rho_1, Q_1, T_2,$

$R_2, (2Q)_1, P_1, (2SM)_2, M_3, L_2, (2MK)_3, K_2,$

$M_8, (MS)_4$.

The order used need not be the same as listed, but whatever order is chosen must be consistent among all the applicable input information. No provision is made for elimination of component effects.

```
C
C
C                         OCEANOGRAPHIC DIVISION
C                          NATIONAL OCEAN SURVEY
C               NATIONAL OCEANIC AND ATMOSPHERIC ADMINISTRATION
C                          6001 EXECUTIVE BLVD
C                       ROCKVILLE, MARYLAND 20852
C                       UNITED STATES OF AMERICA
C
C     PROGRAM LSQHA (INPUT,OUTPUT,TAPE2)
C
C
C  UPDATED  3/21/80  BY HOWARD KUSHNER  ANALYSIS BRANCH
C
C
      IMPLICIT REAL*8(A,B,C,E,P,Q,R,T,V,Y)
      REAL*8 DRAD,F,FR,FREQ,HANG,HSUBN
      REAL*8 S,SETS,SIG,SPEDE,SPEED,SR,SX
      REAL TH
      DIMENSION ID1(20),ID2(20),ID3(20),ID4(20)
       COMMON SINE(5150),TH(9000),PP(90,90),QQ(90,90),THETA(43),
     1 V(90),SPEED(43),FREQ(43),CFREQ(43),SX(90),SIG(90),YEPOCH(43),
     2 YNODE(43),JDAY(14),DAY(14),HSUBN(43),EPOCH(43),COLA(43),P(43),
     3 LOSTVR(87),FMT(6),NCNST(43),R(90),PQP(86),QQQ(86),AMP(43),S(43),
     4 ANG(43),CANG(43),QSX(90),NXS,MAXVRB,NVP2,NV,NVM1,NVP1,PI,TWOPI,
     5 CONV,CUTOFF,NCONST,NSTN,JANEN,JAP,NPT,NOP,MAY,MANDY,IND,MORA,JIM,
     6 JILL,NSR,SR,FR,MORE,IWIND,NDAYS,JMM,JDD,JHH,JX,JZZ,JYY,JVV,JSS,
     7 JFF,JRR,JCF,KAYO2,AA,CORR,RDVR,ESS,ALL,NPIOV2,NTWOPI,NHOURS,SETS,
     8 VH,KKK,HANG,HOUR,C1,C2,IC1,NC(43),SPEDE(43),ID1,ID2,ID3,ID4
   10  READ 180, CUTOFF,NCONST,NSTN,JANEN,JAP,NPT,NOP,MAY,MANDY,IND
       READ 181, JZZ,JYY,JVV,JSS,JFF,JRR,JCF
   20  NXS = 2 * NCONST
       MAXVRB = NXS + NSTN
   25  NVP2 = MAXVRB + 2
       PI = 3.14159265358979
       TWOPI = 2. * PI
       CONV = 180./PI
   35  CALL SINTAB
   38  GO TO (42,40),JYY
   40  MAYD = MAY + 2
       PRINT 182, (SINE(J), J = 1,MAYD)
   42  READ 183, (SPEED(I), I = 1,NCONST)
       DO   44 J = 1,NCONST
   44  FREQ(J) = SPEED(J)/HANG
   45  READ 184, JDAY
       DO 55 K = 1,14
   55  DAY(K) = JDAY(K)
       READ 185, (YEPOCH(I), I = 1,NCONST)
       READ 186, (YNODE(I), I = 1,NCONST)
       READ 187, (P(I), I = 1,NCONST)
       READ 188, C1,C2
```

```
105   PRINT 190
      READ 189, ID1
      READ 189, ID2
      READ 189, ID3
120   DO 130 I = 1,NVP2
      SX(I) = 0.
      DO 125 J = 1,MAXVRB
125   PP(I,J) = 0.
130   CONTINUE
135   READ 189, ID4
      CALL REDATA
      CALL MODT
      PRINT 190
139   GO TO (135,140,150,146,146,146), MORE
140   PRINT 194, (SX(J), J = 1,MAXVRB)
      GO TO 135
146   MORE = MORE - 3
      CALL TRGSA
      GO TO 139
150   CALL CSTAT
155   NVP1 = NV + 1
      NV = NXS + 1
      NVM1 = NV - 1
160   DO 165 L = 1,NVM1
      KPB = L
165   PP(L,NVP1) = KPB
      GO TO (169,166),JCF
166   DO 167 L = 1,NVP1
167   PRINT 195, (PP(L,M), M = 1,NVP1)
169   ALL = 0.
      CALL SCREEN
      CALL ENDPRB
170   GO TO (10,120,160,171,175,135,105,45), MORA
171   READ 181, MORA
      GO TO 160
175   PRINT 196
      STOP
180   FORMAT (F8.7,9I4)
181   FORMAT (7I2)
182   FORMAT (///12H SINE TABLE///(12F10.6))
183   FORMAT (7F10.7)
184   FORMAT (14I3)
185   FORMAT (18F4.1)
186   FORMAT (18F4.3)
187   FORMAT (37F2.0)
188   FORMAT (2F6.2)
189   FORMAT (20A4)
190   FORMAT (2H1 )
194   FORMAT (//6H SUMS //(10F12.1))
195   FORMAT (//8H MATRIX //(12F11.6))
196   FORMAT (////42H END OF OUTPUT FOR THIS JOB      THANK YOU)
```

```
      END
      SUBROUTINE SINTAB
     IMPLICIT REAL*8(A,B,C,E,P,Q,R,T,V,Y)
     REAL*8 DRAD,F,FR,FREQ,HANG,HSUBN
     REAL*8 S,SETS,SIG,SPEDE,SPEED,SR,SX
     REAL TH
      COMMON SINE(5150),TH(9000),PP(90,90),QQ(90,90),THETA(43),
     1 V(90),SPEED(43),FREQ(43),CFREQ(43),SX(90),SIG(90),YEPOCH(43),
     2 YNODE(43),JDAY(14),DAY(14),HSUBN(43),EPOCH(43),COLA(43),P(43),
     3 LOSTVR(87),FMT(6),NCNST(43),R(90),PQP(86),QQQ(86),AMP(43),S(43),
     4 ANG(43),CANG(43),QSX(90),NXS,MAXVRB,NVP2,NV,NVM1,NVP1,PI,TWOPI,
     5 CONV,CUTOFF,NCONST,NSTN,JANEN,JAP,NPT,NOP,MAY,MANDY,IND,MORA,JIM,
     6 JILL,NSR,SR,FR,MORE,IWIND,NDAYS,JMM,JDD,JHH,JX,JZZ,JYY,JVV,JSS,
     7 JFF,JRR,JCF,KAYO2,AA,CORR,RDVR,ESS,ALL,NPIOV2,NTWOPI,NHOURS,SETS,
     8 VH,KKK,HANG,HOUR,C1,C2,IC1,NC(43),SPEDE(43)
   10 NMAY = MAY/MANDY
      MAX = MAY + 1
      MAY2 = 2*MAY
      MAY4 = 4 * MAY
      NPIOV2 = MAY
   15 NTWOPI = MAY4
      AMAY = MAY
      H = PI / (2. * AMAY)
      HANG = 90. / AMAY
   20 B = 2. - H * H
      MART = MANDY
      NART = 0
      JXZ = 1
   25 DO   35 I = 1,NMAY
      NART1 = NART + 1
      NART2 = NART + 2
      PART = NART
      PHIA = PART*H
      PHIB = PHIA+H
      SINE(NART1) = SIN(PHIA)
      SINE(NART2) = SIN(PHIB)
      MART1 = MART - 1
      DO   30 J = NART2,MART1
   30 SINE(J+1) = B * SINE(J) - SINE(J-1)
      NART = MART
      MART = MART + MANDY
      GO TO (35,40),JXZ
   35 CONTINUE
      MART = MAY
      JXZ = 2
      GO TO 25
   40 SINE(MAX) = 1.0
      DO   45 J = 1,MAY
      K = MAX + J
      L = MAX - J
   45 SINE(K) = SINE(L)
```

```
      DO   50 J = 1,MAY2
      K = MAY2 + J
   50 SINE(K) = SINE(J) * (-1.0)
      DO   55 J = 1,MAX
      K = MAY4 + J
   55 SINE(K) = SINE(J)
      RETURN
      END
      SUBROUTINE REDATA
     IMPLICIT REAL*8(A,B,C,E,P,Q,R,T,V,Y)
     REAL*8 DRAD,F,FR,FREQ,HANG,HSUBN
     REAL*8 S,SETS,SIG,SPEDE,SPEED,SR,SX
     REAL TH
      COMMON SINE(5150),TH(9000),PP(90,90),QQ(90,90),THETA(43),
     1 V(90),SPEED(43),FREQ(43),CFREQ(43),SX(90),SIG(90),YEPOCH(43),
     2 YNODE(43),JDAY(14),DAY(14),HSUBN(43),EPOCH(43),COLA(43),P(43),
     3 LOSTVR(87),FMT(6),NCNST(43),R(90),PQP(86),QQQ(86),AMP(43),S(43),
     4 ANG(43),CANG(43),QSX(90),NXS,MAXVRB,NVP2,NV,NVM1,NVP1,PI,TWOPI,
     5 CONV,CUTOFF,NCONST,NSTN,JANEN,JAP,NPT,NOP,MAY,MANDY,IND,MORA,JIM,
     6 JILL,NSR,SR,FR,MORE,IWIND,NDAYS,JMM,JDD,JHH,JX,JZZ,JYY,JVV,JSS,
     7 JFF,JRR,JCF,KAYO2,AA,CORR,RDVR,ESS,ALL,NPIOV2,NTWOPI,NHOURS,SETS,
     8 VH,KKK,HANG,HOUR,C1,C2,IC1,NC(43),SPEDE(43)
   10 READ 101, MORA,JILL,JIM,NSR,SR,FR,MORE,IWIND,NDAYS,JMM,JDD,JHH,JX
      GO TO (25,20), JRR
   20 PRINT 105, CUTOFF,NCONST,NSTN,JANEN,JAP,NPT,NOP,MAY,MANDY,IND,
     1 JZZ,JYY,JVV,JSS,JFF,JRR,JCF,MORA,JILL,JIM,NSR,SR,FR,MORE,IWIND,
     2 NDAYS,JMM,JDD,JHH,JX
   25 GO TO (45,30), JFF
   30 PRINT 110, (YNODE(I), I = 1,NCONST)
      PRINT 115, (YEPOCH(J), J = 1,NCONST)
      PRINT 120, JDAY
   45 READ 125, FMT,IEYE,NHOURS
   47 IF (IWIND) 60,60,50
   50 DO 55 I = 1,IWIND
   55 READ (2,FMT) TRASH
   60 GO TO (80,65),JILL
   65 READ FMT, (TH(I), I = 1,IEYE)
      RETURN
   80 JSR = SR
      NHOURS = NDAYS * 24 * JSR
      NOBS = NHOURS*NSTN
      GO TO (85,90), JZZ
   85 READ (2,FMT) (TH(I), I = 1,NOBS)
      RETURN
   90 READ FMT, (TH(I), I = 1,NOBS)
  100 RETURN
  101 FORMAT (4I2,2F4.0,7I4)
  105 FORMAT (///16H MASTER CONTROL //F8.7,9I6,///15H PRINT CONTROL //
     1 7I5,///14H DATA CONTROL //4I5,2F6.1,7I6)
  110 FORMAT (///13H NODE FACTORS//(10F9.3))
  115 FORMAT (///32H GREENWICH EQUILIBRIUM ARGUMENTS//(10F9.3))
```

```
  120 FORMAT (///9H JDAY(K) //14I6)
  125 FORMAT (15A4,2I4)
      END
      SUBROUTINE MODT
     IMPLICIT REAL*8(A,B,C,E,P,Q,R,T,V,Y)
     REAL*8 DRAD,F,FR,FREQ,HANG,HSUBN
     REAL*8 S,SETS,SIG,SPEDE,SPEED,SR,SX
     REAL TH
      COMMON SINE(5150),TH(9000),PP(90,90),QQ(90,90),THETA(43),
     1 V(90),SPEED(43),FREQ(43),CFREQ(43),SX(90),SIG(90),YEPOCH(43),
     2 YNODE(43),JDAY(14),DAY(14),HSUBN(43),EPOCH(43),COLA(43),P(43),
     3 LOSTVR(87),FMT(6),NCNST(43),R(90),PQP(86),QQQ(86),AMP(43),S(43),
     4 ANG(43),CANG(43),QSX(90),NXS,MAXVRB,NVP2,NV,NVM1,NVP1,PI,TWOPI,
     5 CONV,CUTOFF,NCONST,NSTN,JANEN,JAP,NPT,NOP,MAY,MANDY,IND,MORA,JIM,
     6 JILL,NSR,SR,FR,MORE,IWIND,NDAYS,JMM,JDD,JHH,JX,JZZ,JYY,JVV,JSS,
     7 JFF,JRR,JCF,KAYO2,AA,CORR,RDVR,ESS,ALL,NPIOV2,NTWOPI,NHOURS,SETS,
     8 VH,KKK,HANG,HOUR,C1,C2,IC1,NC(43),SPEDE(43)
   10 SNHOUR = NHOURS/NSR
      SX(NVP2) = SX(NVP2) + SNHOUR
      ANSR = NSR
      GO TO (20,20,20,30),JIM
   20 JHAND = JMM + 1
      HOUR = (JDAY(JHAND) + JDD) * 24 + JHH
      CHOUR = HOUR * SR - ANSR
      DO   25 J = 1,NCONST
      BFREQ = FREQ(J) / SR
      THETA(J) = BFREQ * CHOUR + 1.5
   25 CFREQ(J) = BFREQ * ANSR
   30 DO 125 I = 1,NHOURS,NSR
      GO TO (60,35,45,55),JIM
   35 NNSTN = (I-1)*NSTN
      DO 40 J = 1,NSTN
      K = NXS + J
      L = NNSTN+J
   40 V(K) = TH(L)
      GO TO 65
   45 DO 50 J = 1,NSTN
      JNOBS = (J-1)*NHOURS + I
      K = NXS + J
   50 V(K) = TH(JNOBS)
      GO TO 65
   55 L = (I-1) * 5 + 1
      V(NXS + 1) = TH(L)
      JHAND = TH(L+1) + 1.
      HOUR = (DAY(JHAND) + TH(L+2)) * 24. + TH(L+3) + TH(L+4)/FR
      GO TO 65
   60 V(NXS+1) = TH(I)
   65 DO 100 J = 1,NXS,2
      N = (J + 1) / 2
      GO TO (75,75,75,70),JIM
   70 THETA(N) = HOUR * FREQ(N) + 1.5
```

```
      GO TO 80
   75 THETA(N) = THETA(N) + CFREQ(N)
   80 JTHETA = THETA(N)
      IF (JTHETA) 85,96,90
   85 JTHETA = JTHETA * (-1) + 1
      JTHETA = MOD(JTHETA,NTWOPI)
      JTHETA = NTWOPI - JTHETA
      GO TO 95
   90 JTHETA = MOD(JTHETA,NTWOPI)
   95 IF (JTHETA.NE.0) GO TO 97
   96 JTHETA = NTWOPI
   97 V(J) = SINE(JTHETA)
      K = JTHETA+NPIOV2
      V(J+1) = SINE(K)
  100 CONTINUE
      GO TO (116,105),JVV
  105 PRINT 135, (V(J), J = 1,MAXVRB)
      IF (I.GT.10) GO TO 116
      PRINT 140, (THETA(J), J = 1,NCONST)
  116 IF (JX.EQ.0) GO TO 120
      CALL CPROD2
      GO TO 125
  120 CALL CPROD1
  125 CONTINUE
  130 RETURN
  135 FORMAT (//6H V(J) //(12F10.5))
  140 FORMAT (//10H THETA(J) //(6F16.5))
      END
      SUBROUTINE CPROD1
     IMPLICIT REAL*8(A,B,C,E,P,Q,R,T,V,Y)
     REAL*8 DRAD,F,FR,FREQ,HANG,HSUBN
     REAL*8 S,SETS,SIG,SPEDE,SPEED,SR,SX
     REAL TH
      COMMON SINE(5150),TH(9000),PP(90,90),QQ(90,90),THETA(43),
     1 V(90),SPEED(43),FREQ(43),CFREQ(43),SX(90),SIG(90),YEPOCH(43),
     2 YNODE(43),JDAY(14),DAY(14),HSUBN(43),EPOCH(43),COLA(43),P(43),
     3 LOSTVR(87),FMT(6),NCNST(43),R(90),PQP(86),QQQ(86),AMP(43),S(43),
     4 ANG(43),CANG(43),QSX(90),NXS,MAXVRB,NVP2,NV,NVM1,NVP1,PI,TWOPI,
     5 CONV,CUTOFF,NCONST,NSTN,JANEN,JAP,NPT,NOP,MAY,MANDY,IND,MORA,JIM,
     6 JILL,NSR,SR,FR,MORE,IWIND,NDAYS,JMM,JDD,JHH,JX,JZZ,JYY,JVV,JSS,
     7 JFF,JRR,JCF,KAYO2,AA,CORR,RDVR,ESS,ALL,NPIOV2,NTWOPI,NHOURS,SETS,
     8 VH,KKK,HANG,HOUR,C1,C2,IC1,NC(43),SPEDE(43)
  103 DO 107 J = 1,MAXVRB
  104 SX(J) = SX(J)+V(J)
  105 DO 106 K = J,MAXVRB
  106 PP(K,J) = PP(K,J)+V(J)*V(K)
  107 CONTINUE
      RETURN
      END
      SUBROUTINE CPROD2
     IMPLICIT REAL*8(A,B,C,E,P,Q,R,T,V,Y)
```

```
      REAL*8 DRAD,F,FR,FREQ,HANG,HSUBN
      REAL*8 S,SETS,SIG,SPEDE,SPEED,SR,SX
      REAL TH
       COMMON SINE(5150),TH(9000),PP(90,90),QQ(90,90),THETA(43),
     1 V(90),SPEED(43),FREQ(43),CFREQ(43),SX(90),SIG(90),YEPOCH(43),
     2 YNODE(43),JDAY(14),DAY(14),HSUBN(43),EPOCH(43),COLA(43),P(43),
     3 LOSTVR(87),FMT(6),NCNST(43),R(90),PQP(86),QQQ(86),AMP(43),S(43),
     4 ANG(43),CANG(43),QSX(90),NXS,MAXVRB,NVP2,NV,NVM1,NVP1,PI,TWOPI,
     5 CONV,CUTOFF,NCONST,NSTN,JANEN,JAP,NPT,NOP,MAY,MANDY,IND,MORA,JIM,
     6 JILL,NSR,SR,FR,MORE,IWIND,NDAYS,JMM,JDD,JHH,JX,JZZ,JYY,JVV,JSS,
     7 JFF,JRR,JCF,KAYO2,AA,CORR,RDVR,ESS,ALL,NPIOV2,NTWOPI,NHOURS,SETS,
     8 VH,KKK,HANG,HOUR,C1,C2,IC1,NC(43),SPEDE(43)
       NNXS = NXS + 1
       DO 105 J = NNXS,MAXVRB
  105  SX(J) = SX(J) + V(J)
       DO 107 K = 1,MAXVRB
       DO 107 J = NNXS,MAXVRB
  107  PP(J,K) = PP(J,K) + V(J) * V(K)
       RETURN
       END
       SUBROUTINE CSTAT
      IMPLICIT REAL*8(A,B,C,E,P,Q,R,T,V,Y)
      REAL*8 DRAD,F,FR,FREQ,HANG,HSUBN
      REAL*8 S,SETS,SIG,SPEDE,SPEED,SR,SX
      REAL TH
       COMMON SINE(5150),TH(9000),PP(90,90),QQ(90,90),THETA(43),
     1 V(90),SPEED(43),FREQ(43),CFREQ(43),SX(90),SIG(90),YEPOCH(43),
     2 YNODE(43),JDAY(14),DAY(14),HSUBN(43),EPOCH(43),COLA(43),P(43),
     3 LOSTVR(87),FMT(6),NCNST(43),R(90),PQP(86),QQQ(86),AMP(43),S(43),
     4 ANG(43),CANG(43),QSX(90),NXS,MAXVRB,NVP2,NV,NVM1,NVP1,PI,TWOPI,
     5 CONV,CUTOFF,NCONST,NSTN,JANEN,JAP,NPT,NOP,MAY,MANDY,IND,MORA,JIM,
     6 JILL,NSR,SR,FR,MORE,IWIND,NDAYS,JMM,JDD,JHH,JX,JZZ,JYY,JVV,JSS,
     7 JFF,JRR,JCF,KAYO2,AA,CORR,RDVR,ESS,ALL,NPIOV2,NTWOPI,NHOURS,SETS,
     8 VH,KKK,HANG,HOUR,C1,C2,IC1,NC(43),SPEDE(43)
   10  NV = MAXVRB
       SETS = SX(NVP2)
       GO TO (19,19,19,11,19,11,19,19), MORA
   11  DO 15 J = 1,MAXVRB
       DO 12 K = J,MAXVRB
   12  QQ(K,J) = PP(K,J)
       QSX(J) = SX(J)
   15  CONTINUE
   19  DO 20 I = 1,NV
   20  SX(I) = SX(I) / SETS
       DO 40 I = 1,NV
       DO 30 J = 1,NV
   30  PP(J,I) = PP(J,I) - SETS * SX(I) * SX(J)
   40  CONTINUE
       DO 50 M = 1,NV
   50  SIG(M) = SQRT(PP(M,M) / SETS)
       PRINT 165, SETS
```

```
      PRINT 170, (SX(I), I = 1,NV)
      GO TO (105,80), JSS
   80 PRINT 175, (SIG(M), M = 1,NV)
      PRINT 180
  105 DO 110 L = 2,NV
      DO 110 M = 2,L
  110 PP(M-1,L) = PP(L,M-1)
      IF (MORA.NE.3) GO TO 125
  115 DO 120 L = 1,NV
      DO 120 M = 1,NV
  120 QQ(L,M) = PP(L,M)
  125 DO 160 K = 1,NV
      DO 130 KC = K,NV
  130 R(KC) = PP(K,KC) / SIG(K) / SIG(KC) / SETS
      GO TO (160,140), JSS
  140 PRINT 185, K,(R(KC), KC = K,NV)
  160 CONTINUE
      RETURN
  165 FORMAT (//7H SETS =,F5.0)
  170 FORMAT (//17H LISTING OF MEANS//(10F12.4))
  175 FORMAT (//31H LISTING OF STANDARD DEVIATIONS//(10F12.4))
  180 FORMAT (//36H LISTING OF CORRELATION COEFFICIENTS)
  185 FORMAT (/9H VARIABLE,I4//(10F12.4))
      END
      SUBROUTINE SCREEN
     IMPLICIT REAL*8(A,B,C,E,P,Q,R,T,V,Y)
     REAL*8 DRAD,F,FR,FREQ,HANG,HSUBN
     REAL*8 S,SETS,SIG,SPEDE,SPEED,SR,SX
     REAL TH
      COMMON SINE(5150),TH(9000),PP(90,90),QQ(90,90),THETA(43),
     1 V(90),SPEED(43),FREQ(43),CFREQ(43),SX(90),SIG(90),YEPOCH(43),
     2 YNODE(43),JDAY(14),DAY(14),HSUBN(43),EPOCH(43),COLA(43),P(43),
     3 LOSTVR(87),FMT(6),NCNST(43),R(90),PQP(86),QQQ(86),AMP(43),S(43),
     4 ANG(43),CANG(43),QSX(90),NXS,MAXVRB,NVP2,NV,NVM1,NVP1,PI,TWOPI,
     5 CONV,CUTOFF,NCONST,NSTN,JANEN,JAP,NPT,NOP,MAY,MANDY,IND,MORA,JIM,
     6 JILL,NSR,SR,FR,MORE,IWIND,NDAYS,JMM,JDD,JHH,JX,JZZ,JYY,JVV,JSS,
     7 JFF,JRR,JCF,KAYO2,AA,CORR,RDVR,ESS,ALL,NPIOV2,NTWOPI,NHOURS,SETS,
     8 VH,KKK,HANG,HOUR,C1,C2,IC1,NC(43),SPEDE(43)
      DO 10 K = 1,NVM1
      PQP(K) = 0.0
      QQQ(K) = 0.0
   10 CONTINUE
      DO 225 K = 1,NVM1,2
      JAM = 1
      VH = 0.
   15 DO  45 L = K,NVM1,2
   20 IF (PP(L,L).NE.0.0) GO TO 25
      PQP(L) = PP(L,NVP1)
      PP(L,L) = 0.000001
   25 IF (PP(L+1,L+1).NE.0.0) GO TO 35
      PQP(L+1) = PP(L+1,NVP1)
```

```
      PP(L+1,L+1) = 0.000001
   35 TVH = ((PP(L,NV)**2)/(PP(L,L)*PP(NV,NV))+(PP(L+1,NV)**2)/(PP(L+1,
     1 L+1)*PP(NV,NV))+2.*PP(L,L+1)*PP(L,NV)*PP(L+1,NV)/(PP(L,L)*PP(L+1,
     2 L+1)*PP(NV,NV)))/(1.-(PP(L,L+1)**2)/(PP(L,L)*PP(L+1,L+1)))
   40 IF (TVH.GE.0.0) GO TO 44
      TVH = -1.0 * TVH
   44 IF (VH.GT.TVH) GO TO 45
      VH = TVH
      KEY = L
   45 CONTINUE
   50 IF (VH.GE.CUTOFF) GO TO 55
      KL = K
      KX=K-1
      KKK = K - 2
      ALL = 1.
      GO TO 155
   55 DO 60 M = 1,NV
      TVH = PP(M,K)
      PP(M,K) = PP(M,KEY)
   60 PP(M,KEY) = TVH
      DO 65 M = 1,NV
      TVH = PP(M,K+1)
      PP(M,K+1) = PP(M,KEY+1)
   65 PP(M,KEY+1) = TVH
      DO 70 M = 1,NVP1
      TVH = PP(K,M)
      PP(K,M) = PP(KEY,M)
   70 PP(KEY,M) = TVH
      DO 75 M = 1,NVP1
      TVH = PP(K+1,M)
      PP(K+1,M) = PP(KEY+1,M)
   75 PP(KEY+1,M) = TVH
      KAY = K
   80 IF (PP(KAY,KAY)) 90,85,90
   85 QQQ(KAY) = PP(KAY,NVP1)
      PP(KAY,KAY) = 0.000001
   90 DO 105 L = 1,NV
      IF (L-KAY) 95,105,95
   95 F = PP(L,KAY)/PP(KAY,KAY)
      DO  100 M = KAY,NV
  100 PP(L,M) = PP(L,M) - F*PP(KAY,M)
  105 CONTINUE
  110 F = PP(KAY,KAY)
  115 DO  120 M = KAY,NV
  120 PP(KAY,M) = PP(KAY,M)/F
      GO TO (125,130),JAM
  125 JAM = 2
      KAY = KAY+1
      GO TO 80
  130 IF (PP(NV,NV)) 132,132,131
  131 ESS = SQRT (PP(NV,NV) / SETS)
```

```
      RDVR = 1. - (ESS/SIG(NV))**2
132   KAY02 = KAY / 2
      IF(K.EQ.(NXS-1)) GO TO 154
      IF (NOP.GT.KAY02) GO TO 154
      IF(NOP.LT.KAY02) GO TO 145
135   PRINT 256
145   KATHY = PP(KAY,NVP1)/2.0
      PRINT 257, KAY02,KATHY,RDVR
      GO TO 225
154   KX = K+1
      KKK = K
155   AA = SX(NV)
160   DO 165 N = 1,KX
      NX = PP(N,NVP1)
165   AA = AA - PP(N,NV) * SX(NX)
      CORR = SQRT(RDVR)
      IC1 = K
170   CALL SPRNT1
175   GO TO (180,215),JAP
180   J = 1
      DO 212 N = 1,KKK,2
185   HSUBN(J) = SQRT(PP(N,NV)**2 + PP(N+1,NV)**2)
      IF (PP(N+1,NV)) 186,187,186
186   DRAD = ATAN(PP(N,NV) / PP(N+1,NV))
      GO TO 190
187   IF (PP(N,NV)) 188,210,189
188   EPOCH(J) = 270.0
      GO TO 210
189   EPOCH(J) = 90.0
      GO TO 210
190   IF (PP(N,NV).EQ.(0.0).AND.PP(N+1,NV).GT.(0.0)) GO TO 195
      IF (PP(N,NV).GE.(0.0).AND.DRAD.LE.(0.0)) GO TO 200
      IF (PP(N,NV).LT.(0.0).AND.DRAD.GT.(0.0)) GO TO 200
      IF (PP(N,NV).LT.(0.0).AND.DRAD.LT.(0.0)) GO TO 205
195   EPOCH(J) = DRAD * CONV
      GO TO 210
200   EPOCH(J) = (DRAD + PI) * CONV
      GO TO 210
205   EPOCH(J) = (DRAD + TWOPI) * CONV
210   NCNST(J) = PP(N+1,NVP1) / 2.
      J = J + 1
212   CONTINUE
      CALL SPRNT2
215   IF (ALL.NE.0.) GO TO 235
      IF (KAY02.EQ.NPT)  GO TO 255
225   CONTINUE
230   IF(KKK.EQ.(NV-2)) GO TO 241
235   PRINT 258, (PP(I,NVP1), I = KL,NVM1)
241   PRINT 259, (PQP(I), I = 1,NVM1)
      PRINT 260, (QQQ(I), I = 1,NVM1)
255   RETURN
```

```
  256 FORMAT (//56H MULTIPLE CORRELATION COEFFICIENT WITH (N) CONSTITUE
     1NTS.)
  257 FORMAT (//6H  N = ,I3,11H  CON. NO. ,I3,30H  TOTAL REDUCTION OF V
     1ARIANCE ,F7.4)
  258 FORMAT (18H DROPPED VARIABLE ,/12F6.0)
  259 FORMAT (//37H VARIABLES WITH PP(L,L) EQUAL TO 0.0 //(20F4.0))
  260 FORMAT (//39H VARIABLES WITH PP(L,L) SET = 0.000001 //(20F4.0))
      END
      SUBROUTINE SPRNT1
     IMPLICIT REAL*8(A,B,C,E,P,Q,R,T,V,Y)
     REAL*8 DRAD,F,FR,FREQ,HANG,HSUBN
     REAL*8 S,SETS,SIG,SPEDE,SPEED,SR,SX
     REAL TH
      COMMON SINE(5150),TH(9000),PP(90,90),QQ(90,90),THETA(43),
     1 V(90),SPEED(43),FREQ(43),CFREQ(43),SX(90),SIG(90),YEPOCH(43),
     2 YNODE(43),JDAY(14),DAY(14),HSUBN(43),EPOCH(43),COLA(43),P(43),
     3 LOSTVR(87),FMT(6),NCNST(43),R(90),PQP(86),QQQ(86),AMP(43),S(43),
     4 ANG(43),CANG(43),QSX(90),NXS,MAXVRB,NVP2,NV,NVM1,NVP1,PI,TWOPI,
     5 CONV,CUTOFF,NCONST,NSTN,JANEN,JAP,NPT,NOP,MAY,MANDY,IND,MORA,JIM,
     6 JILL,NSR,SR,FR,MORE,IWIND,NDAYS,JMM,JDD,JHH,JX,JZZ,JYY,JVV,JSS,
     7 JFF,JRR,JCF,KAYO2,AA,CORR,RDVR,ESS,ALL,NPIOV2,NTWOPI,NHOURS,SETS,
     8 VH,KKK,HANG,HOUR,C1,C2,IC1,NC(43),SPEDE(43)
   10 GO TO (20,40),JAP
   20 ICON = PP(IC1+1,NVP1)/2.
      PRINT 91, ICON,VH
      PRINT 93, CORR,RDVR,ESS
      RETURN
   40 PRINT 92, PP(KKK,NVP1),VH
      PRINT 93, CORR,RDVR,ESS
      PRINT 94, AA,(PP(N,NV),PP(N,NVP1), N = 1,KKK)
      RETURN
   91 FORMAT (///14H CONSTITUENT (,I2,31H) REDUCES RESIDUAL VARIANCE BY
     1 ,F10.7)
   92 FORMAT (///11H VARIABLE (,F3.0,21H) REDUCES VARIANCE BY,F10.7)
   93 FORMAT (/23H MULTIPLE CORR. COEF. =,F7.4//30H TOTAL REDUCTION OF
     1VARIANCE =,F7.4//30H STANDARD DEVIATION OF ERROR =,F9.4//)
   94 FORMAT (5H Y  =,F19.6,4(3H  +,F11.6,2H (,F4.0,1H))/5(3H  +,F11.6,
     1 2H (,F4.0,1H)))
      END
      SUBROUTINE SPRNT2
     IMPLICIT REAL*8(A,B,C,E,P,Q,R,T,V,Y)
     REAL*8 DRAD,F,FR,FREQ,HANG,HSUBN
     REAL*8 S,SETS,SIG,SPEDE,SPEED,SR,SX
     REAL TH
      COMMON SINE(5150),TH(9000),PP(90,90),QQ(90,90),THETA(43),
     1 V(90),SPEED(43),FREQ(43),CFREQ(43),SX(90),SIG(90),YEPOCH(43),
     2 YNODE(43),JDAY(14),DAY(14),HSUBN(43),EPOCH(43),COLA(43),P(43),
     3 LOSTVR(87),FMT(6),NCNST(43),R(90),PQP(86),QQQ(86),AMP(43),S(43),
     4 ANG(43),CANG(43),QSX(90),NXS,MAXVRB,NVP2,NV,NVM1,NVP1,PI,TWOPI,
     5 CONV,CUTOFF,NCONST,NSTN,JANEN,JAP,NPT,NOP,MAY,MANDY,IND,MORA,JIM,
     6 JILL,NSR,SR,FR,MORE,IWIND,NDAYS,JMM,JDD,JHH,JX,JZZ,JYY,JVV,JSS,
```

```
     7 JFF,JRR,JCF,KAY02,AA,CORR,RDVR,ESS,ALL,NPIOV2,NTWOPI,NHOURS,SETS,
     8 VH,KKK,HANG,HOUR,C1,C2,IC1,NC(43),SPEDE(43),ID1(20),ID2(20),
     9 ID3(20),ID4(20)
       JANE = JANEN
       L = 1
       DO 30 J = 1,NCONST
       DO 10 N = 1,KAY02
       IF (NCNST(N).EQ.J) GO TO 20
   10  CONTINUE
       GO TO 30
   20  NC(L) = NCNST(N)
       AMP(L) = HSUBN(N)
       ANG(L) = EPOCH(N)
       NSP = NC(L)
       SPEDE(L) = SPEED(NSP)
       IF (L.EQ.KAY02) GO TO 35
       L = L + 1
   30  CONTINUE
   35  PRINT 125, AA,(NC(N),AMP(N),ANG(N),SPEDE(N), N = 1,KAY02)
   75  GO TO (80,120),JANE
   80  DO 115 N = 1,KAY02
       NCS = NC(N)
       AMP(N) = AMP(N) / YNODE(NCS)
       COLA(N) = P(NCS) * C1 - (SPEDE(N) * C2 / 15.)
   81  IF (COLA(N)+360.) 86,88,83
   83  IF (COLA(N)-360.) 90,88,84
   84  COLA(N) = COLA(N) - 360.
       GO TO 83
   86  COLA(N) = COLA(N) + 360.
       GO TO 81
   88  COLA(N) = 0.
   90  CANG(N) = YEPOCH(NCS) + ANG(N)
   91  IF (CANG(N)-360.) 92,93,93
   92  ANG(N) = CANG(N) - COLA(N)
       IF(ANG(N)) 94,115,95
   93  CANG(N) = CANG(N) - 360.
       GO TO 92
   94  ANG(N) = ANG(N) + 360.
       GO TO 115
   95  IF (ANG(N) - 360.) 115,100,100
  100  ANG(N) = ANG(N) - 360.
  115  CONTINUE
  116  JANE = 2
       PRINT 188
       PRINT 189, ID1
       PRINT 189, ID2
       PRINT 189, ID3
       PRINT 189, ID4
       PRINT 135
       PRINT 130, AA,(NC(N),AMP(N),ANG(N),COLA(N),CANG(N),SPEDE(N), N =
     1 1,KAY02)
```

```
  120 RETURN
  125 FORMAT (6X,47HMSL    CONSTIT.     R          ZETA          SPEED//
     1 F10.3,I7,F12.3,F12.2,F13.4/(15X,I2,F12.3,F12.2,F13.4))
  130 FORMAT(6X,73HMSL    CONSTIT.     H          KAPPA         KPR-K
     1KAPPA PRIME     SPEED//F10.3,I7,F12.3,2F12.2,F12.1,F15.4/(15X,I2,
     2 F12.3,2F12.2,F12.1,F15.4))
  135 FORMAT (//29H   (NODE FACTORS CONSIDERED) )
  188 FORMAT (2H1 )
  189 FORMAT (3X,20A4)
      END
      SUBROUTINE ENDPRB
     IMPLICIT REAL*8(A,B,C,E,P,Q,R,T,V,Y)
     REAL*8 DRAD,F,FR,FREQ,HANG,HSUBN
     REAL*8 S,SETS,SIG,SPEDE,SPEED,SR,SX
     REAL TH
      COMMON SINE(5150),TH(9000),PP(90,90),QQ(90,90),THETA(43),
     1 V(90),SPEED(43),FREQ(43),CFREQ(43),SX(90),SIG(90),YEPOCH(43),
     2 YNODE(43),JDAY(14),DAY(14),HSUBN(43),EPOCH(43),COLA(43),P(43),
     3 LOSTVR(87),FMT(6),NCNST(43),R(90),PQP(86),QQQ(86),AMP(43),S(43),
     4 ANG(43),CANG(43),QSX(90),NXS,MAXVRB,NVP2,NV,NVM1,NVP1,PI,TWOPI,
     5 CONV,CUTOFF,NCONST,NSTN,JANEN,JAP,NPT,NOP,MAY,MANDY,IND,MORA,JIM,
     6 JILL,NSR,SR,FR,MORE,IWIND,NDAYS,JMM,JDD,JHH,JX,JZZ,JYY,JVV,JSS,
     7 JFF,JRR,JCF,KAYO2,AA,CORR,RDVR,ESS,ALL,NPIOV2,NTWOPI,NHOURS,SETS,
     8 VH,KKK,HANG,HOUR,C1,C2,IC1,NC(43),SPEDE(43)
   18 GO TO (20,20,35,61,130,61,125,125), MORA
   20 REWIND 2
      RETURN
   35 IF(IND.LT.MAXVRB) GO TO 40
      READ 135, MORA
      GO TO 18
   40 IND = IND + 1
   45 DO   50 J = 1,MAXVRB
      DO   50 K = 1,MAXVRB
   50 PP(J,K) = QQ(J,K)
   55 DO 58 J = 1,MAXVRB
   58 PP(NV,J) = PP(IND,J)
      DO 60 J = 1,MAXVRB
   60 PP(J,NV) = PP(J,IND)
      SIG(NV) = SIG(IND)
      SX(NV) = SX(IND)
      GO TO 90
   61 DO 64 J = 1,MAXVRB
      DO 63 K = J,MAXVRB
   63 PP(K,J) = QQ(K,J)
      SX(J) = QSX(J)
   64 CONTINUE
      IF (MORA.EQ.6) GO TO 130
   65 READ 140, M
      READ 140, (LOSTVR(I), I = 1,M)
      NV = LOSTVR(1)
      NVP1 = NV + 1
```

```
      NVM1 = NV - 1
   75 DO   85 I = 2,M
      N = LOSTVR(I)
      DO 80 J = 1,MAXVRB
      SX(N) = 0.
      PP(N,J) = 0.
   80 PP(J,N) = 0.
   85 CONTINUE
   90 READ 145
      PRINT 150
      PRINT 145
      RETURN
  125 READ 155, CUTOFF,NCONST,NSTN,JANEN,JAP,NPT,NOP,MAY,MANDY,IND
      NXS = 2 * NCONST
      MAXVRB = NXS + NSTN
      NVP2 = MAXVRB + 2
  130 RETURN
  135 FORMAT (I2)
  140 FORMAT (24I3)
  145 FORMAT (80H
     1
  150 FORMAT (2H1 )
  155 FORMAT (F8.7,9I4)
      END
      SUBROUTINE TRGSA
     IMPLICIT REAL*8(A,B,C,E,P,Q,R,T,V,Y)
     REAL*8 DRAD,F,FR,FREQ,HANG,HSUBN
     REAL*8 S,SETS,SIG,SPEDE,SPEED,SR,SX
     REAL TH
      COMMON SINE(5150),TH(9000),PP(90,90),QQ(90,90),THETA(43),
     1 V(90),SPEED(43),FREQ(43),CFREQ(43),SX(90),SIG(90),YEPOCH(43),
     2 YNODE(43),JDAY(14),DAY(14),HSUBN(43),EPOCH(43),COLA(43),P(43),
     3 LOSTVR(87),FMT(6),NCNST(43),R(90),PQP(86),QQQ(86),AMP(43),S(43),
     4 ANG(43),CANG(43),QSX(90),NXS,MAXVRB,NVP2,NV,NVM1,NVP1,PI,TWOPI,
     5 CONV,CUTOFF,NCONST,NSTN,JANEN,JAP,NPT,NOP,MAY,MANDY,IND,MORA,JIM,
     6 JILL,NSR,SR,FR,MORE,IWIND,NDAYS,JMM,JDD,JHH,JX,JZZ,JYY,JVV,JSS,
     7 JFF,JRR,JCF,KAYO2,AA,CORR,RDVR,ESS,ALL,NPIOV2,NTWOPI,NHOURS,SETS,
     8 VH,KKK,HANG,HOUR,C1,C2,IC1,NC(43),SPEDE(43)
      ANSR = NSR
      IHOUR = HOUR * SR - ANSR
      FHOUR = (IHOUR + NHOURS) / NSR
      H = PI/180.
      DO 10 J = 1,NCONST
      S(J) = SPEED(J) * H / SR * ANSR
   10 CONTINUE
      JXQ = JX
      ALPH = 1.
   12 DO 200 I = 1,NXS,2
      J = (I + 1)/2
      PHI1 = S(J)/2.
      JJJ = 1
```

```
   15 PHI2 = FHOUR*PHI1
      PHI3 = PHI2 + PHI1
      COF = ALPH * SIN(PHI3) / SIN(PHI1)
      GO TO (20,30), JJJ
   20 SX(I) = SX(I) + SIN(PHI2) * COF
      SX(I+1) = SX(I+1) + COS(PHI2) * COF
      PHI1 = S(J)
      JJJ = 2
      GO TO 15
   30 SUMC = COS(PHI2) * COF
      PP(I,I) = PP(I,I) + (ALPH * FHOUR - SUMC) / 2.
      PP(I+1,I) = PP(I+1,I) + (SIN(PHI2) * COF) / 2.
      PP(I+1,I+1) = PP(I+1,I+1) + (ALPH * FHOUR + SUMC) / 2.
      IF (I.GE.(NXS - 1)) GO TO 200
      M = I + 2
      DO 100 K = M,NXS,2
      L = (K+1) / 2
      PHI1 = (S(J) + S(L)) / 2.
      JJJ = 1
      GO TO 50
   40 SCOSP = COS(PHI2) * COF
      SSINP = SIN(PHI2) * COF
      PHI1 = (S(J) - S(L)) / 2.
      JJJ = 2
   50 PHI2 = FHOUR * PHI1
      PHI3 = PHI2 + PHI1
      COF = ALPH * SIN(PHI3) / SIN(PHI1)
      GO TO (40,60), JJJ
   60 SCOSM = COS(PHI2) * COF
      SSINM = SIN(PHI2) * COF
      PP(K,I) = PP(K,I) + (SCOSM - SCOSP) / 2.
      PP(K,I+1) = PP(K,I+1) - (SSINM - SSINP) / 2.
      PP(K+1,I) = PP(K+1,I) + (SSINM + SSINP) / 2.
      PP(K+1,I+1) = PP(K+1,I+1) + (SCOSM + SCOSP) / 2.
  100 CONTINUE
  200 CONTINUE
      GO TO (210,220),JXQ
  210 RETURN
  220 ALPH = -1.
      JXQ = 1
      FHOUR = IHOUR / NSR
      GO TO 12
      END
```

7. APPENDIX B - ZETLER, BERNARD D., 1978, "TIDE PREDICTIONS", ARTICLE PUBLISHED IN GEOPHYSICAL PREDICTIONS, NATIONAL ACADEMY OF SCIENCES.

Tide Predictions

14

BERNARD D. ZETLER
Scripps Institution of Oceanography

14.1 HISTORY

In populated areas where tides are important, local predictions have been published in newspapers for as long as living people remember; their availability is taken as much for granted as times of sunrise and sunset. For the most part, their accuracy is also unquestioned, although the public is aware that during storms and hurricanes "tides" much higher, and occasionally lower, than normal (i.e., predicted) may occur.

Although we know that tides result primarily from the tide-producing forces of the moon and sun acting upon the rotating earth, our knowledge of the dynamics of ocean and estuarine tides needs significant improvement; empirical procedures developed in the nineteenth century make it possible to prepare reasonably good tide predictions for any location provided that tide observations have been obtained at that place in the past and the local sea-floor topography remains unchanged. It is a tribute to many scientists, in particular Sir William Thomson (later Lord Kelvin) and George Howard Darwin in England and William Ferrel and Rollin A. Harris in the United States, that the state of the art in tide predictions reached so high a plateau in the early twentieth century that significant improvements were not achieved for a half-century.

Although the Coast and Geodetic Survey (now called the National Ocean Survey) celebrated its centenary of published tide predictions in 1968, the Coast Survey had been supplying navigators with information from which they could prepare their own predictions as far back as 1844. This was done by including mean high-water lunitidal intervals (the interval between the moon's upper or lower transit over the local meridian and the following high water) and tidal ranges on the Survey's nautical charts. A published method for improving the crude predictions (presumably derived from John Lubbock's research in England in the 1830's) provided corrections for the effects of the phase of the moon and the declination and parallax of the moon and sun. However, despite Figure 14.1, the correlation between some tidal and lunar parameters were known and used long before the time of Newton.

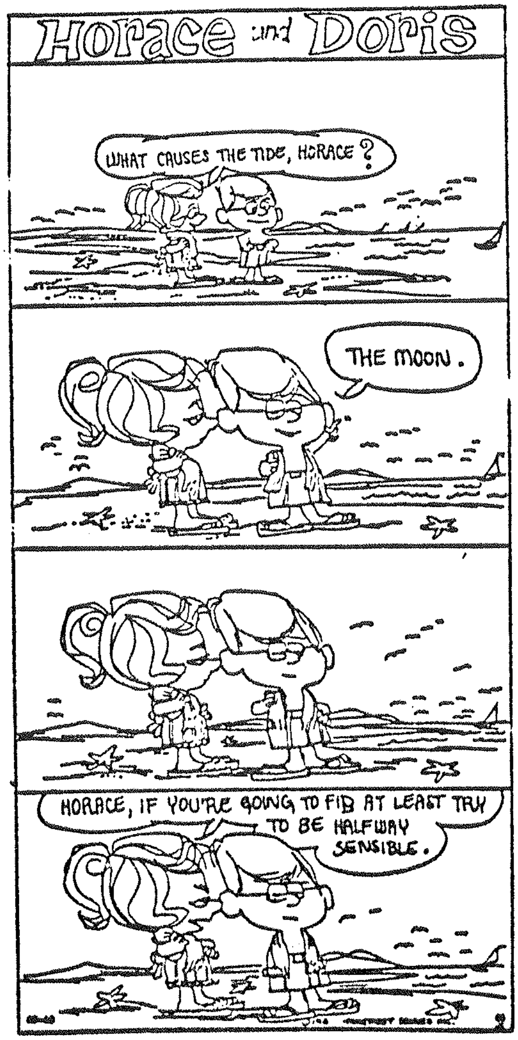

FIGURE 14.1 Cartoon, courtesy of Foremost Foods Company.

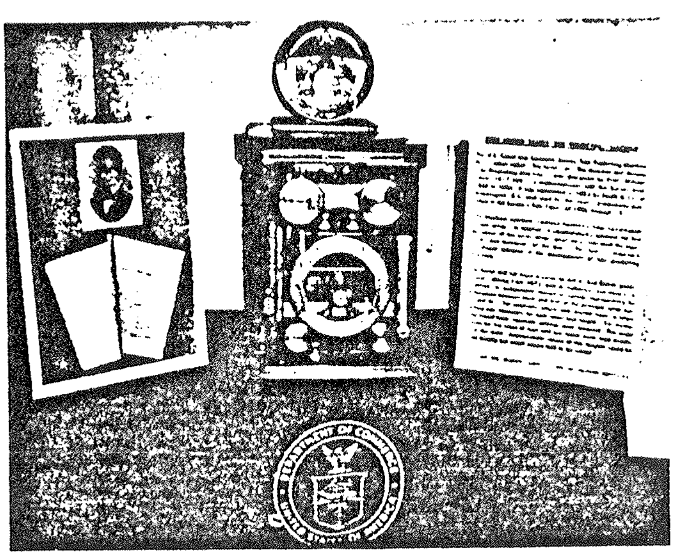

FIGURE 14.2 First U.S. tide-predicting machine, used 1885–1911.

Later in the nineteenth century, Thomson and Darwin developed the harmonic method of analyzing and predicting tides. The method uses sinusoids (tidal constituents) whose frequencies, derived from astronomic observations, are sums and differences of six basic frequencies in the motions of the earth, moon, and sun: the day, month, year, 8.9 years (lunar perigee), 18.6 years (lunar nodes), and 21,000 years (solar perigee). The amplitude and phase lag behind a theoretical (equilibrium) tide for each constituent are obtained by analysis; a prediction synthesizes the same curves for some future period. The distortion of tides as they move into shallow water is simulated by analyzing for frequencies that are harmonics of the fundamental frequencies or combinations of these.

Sir William Thomson made the first tide-predicting machine in 1873 under the auspices of the British Association for the Advancement of Science; it summed 10 constituents and automatically traced a curve showing the predicted heights. This country's first mechanical tide prediction machine (19 constituents) was designed by William Ferrel of the Coast and Geodetic Survey. The machine, introduced in 1885, provided for times and heights of high and low tides to be read directly from the dial indicators but did not produce a plotted prediction (Figure 14.2). An improved mechanical predictor for 37 constituents was completed by R. A. Harris and E. G. Fischer of the Coast and Geodetic Survey in 1910. This machine (Figure 14.3) was the first to compute simultaneously the height of the tide and the times of high and low waters, the machine automatically stopping to allow the information to be read from dials. It also produced a continuous plot with time marks for each hour, maximum and minimum. Although the 37 incommensurable curves summed by the machine range from one cycle per year to eight cycles per lunar day (24.8 h), no constituent accumulated a phase error greater than 2 degrees in a year's predictions, a remarkable achievement in designing and fabricating the gear system.

For many North Sea locations, combinations of the customary shallow-water constituents cannot adequately reproduce the shape of the observed tides. Horn in Germany and Doodson in England developed nonharmonic procedures for empirically correcting predictions to improve the accuracy of published tables.

14.2 USES

Fundamentally, tide predictions are intended to augment in a time sense the depth information available on a nautical chart. The predicted height for a particular place at a given time is added algebraically to the depth shown on the chart to obtain the depth of water at that time. Thus, a navigator uses the tide table to determine whether his vessel can safely pass over a plotted course on the chart. In most countries, tide predictions are prepared by the hydrographic service, frequently a branch of the Navy.

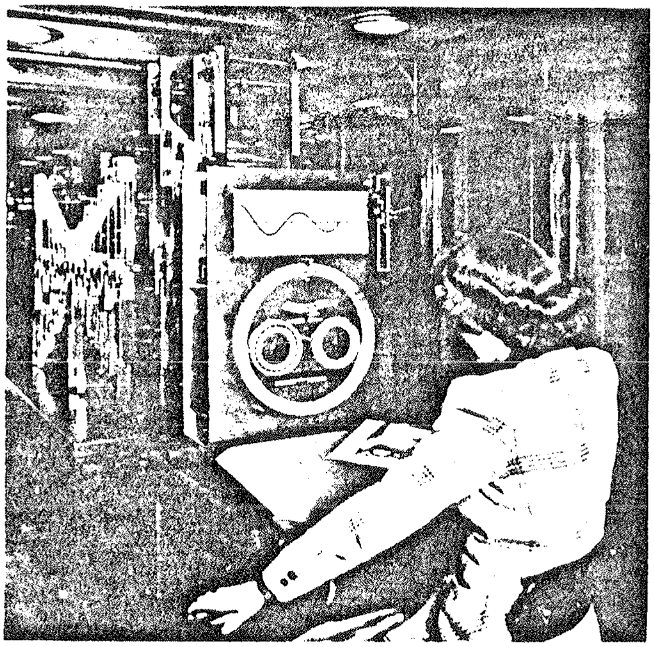

FIGURE 14.3 Second U.S. tide-predicting machine, used 1912–1965.

Tide refers to vertical changes in water level. There are associated horizontal changes in water flow that are referred to as tidal currents in the United States and as tidal streams in the United Kingdom. Tidal current predictions in their simplest form (reversing as for a narrow estuary) provide times of slack and times and speeds of maximum flood and ebb currents. Tidal currents are often rotary in broad estuaries and on the continental shelves, making more complex prediction formats necessary. For navigation purposes, the times of slack water are frequently more important than the times of tide, even for deep-draft vessels.

Tidal motion in the open ocean is generally considered to be a composite of barotropic and baroclinic tides. Barotropic tides can be described as surface waves in which horizontal velocity is constant with depth except in the frictional bottom boundary layer, and vertical velocity increases linearly from the bottom to the surface; baroclinic tides are internal waves at tidal frequencies with vertical and horizontal velocities oscillatory with depth, the maximum vertical displacements occurring within the water column and with very small vertical motion at the free surface. Tidal measurements at the ocean surface and pressure measurements on the sea floor are almost independent of the effect of internal tides and therefore are ordinarily described as barotropic. Current measurements within the water column are vectorial sums of roughly comparable barotropic and baroclinic contributions. Since baroclinic tides are much more variable (in particular not locked in phase with the tide-producing forces as are barotropic tides), ocean tidal currents cannot be predicted as accurately as tidal heights. To compound the problem, tidal current measurements are more difficult to obtain and ordinarily have a poorer signal-to-noise ratio; unlike tides, tidal currents cannot be accurately interpolated from nearby observations.

Tide predictions are vital for some military operations. For amphibious landings it is customary to arrive just before high water to land as high up on the beach as possible but to allow a small margin of safety so that the rising tide will free an amphibious craft that might ground on a reef before reaching shore. Figure 14.4 is a sample of many such predictions prepared during World War II. In another application, it has been reported that some of the most intense fire bombings of Britain during the same war were timed to correspond to extreme low tides so that water pressure for extinguishing the flames would be at a minimum. Large German warships carried portable 12-constituent tide-predicting machines since they were cut off from their home bases and therefore might not receive up-to-date tide tables.

14.3 RECENT CHANGES

Rapid proliferation and improvements in electronic computers in the middle of this century finally ended the long status quo in the state of the art for tide predictions. Although the Coast and Geodetic Survey tide predictor withstood the first assault when predictions on an IBM 650 were found to take longer than on the half-century-old machine, its victory was short lived, and by 1965 it had become a museum piece. A similar change took place at about the same time in England, but Kelvin mechanical tide-predictors may still be in use in some countries.

There were advantages other than speed and efficiency in shifting to electronic computers for tide prediction. As long as the fixed gearing of the mechanical predictor was limited to a finite set of particular constituents, research demonstrating the importance of other constituents had a low priority. Although there were sentimental regrets at replacing such a fine instrument, it was a distinct relief no longer to be dependent on the one machine; it was known to be wearing out, but, even worse, there was a fear of sabotage, particularly during war years.

In addition to the direct application of electronic computers to tide predictions, computer availability made possible more detailed analyses (for example, studies of 50 years of hourly heights, roughly a half million values) and digital recording (thus enhancing the state of the art for tide gauges as well as making possible more tide observations by reducing the manpower needs in data processing). Soon thereafter, dramatic advances were achieved in numerical modeling of tides, permitting for the first time research into the physics of tide generation in the world's oceans. Some of these advances have already contributed to improved predictions, and others will do so in the future.

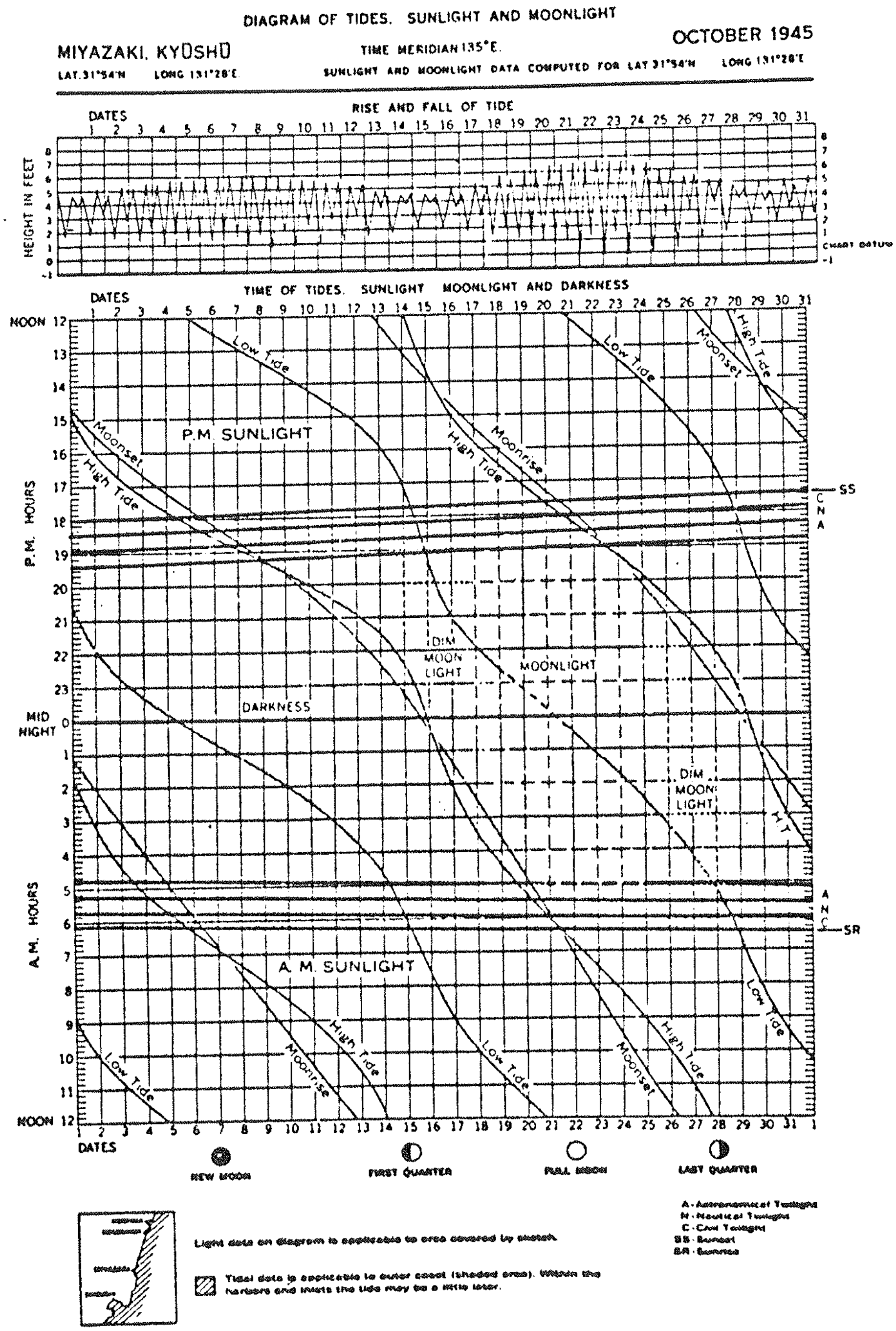

FIGURE 14.4 Tide and light diagram prepared during World War II.

It seems logical to group these recent improvements as follows:

A. *Data Analysis*

1. Change in harmonic analysis procedure. The traditional analysis in the Coast and Geodetic Survey (now the National Ocean Survey) solves for each constituent separately by a procedure that tries to emulate what would be achieved by cutting the marigram (tide record) into constituent periods (for example, every 25.82 h for O_1, a principal lunar diurnal constituent), piling the cut sections vertically and averaging a mean curve for the constituent period through the pile (Schureman, 1941). By means of stencil overlays, hourly heights (solar time) are identified as constituent hours (with errors not exceeding 0.5 h and algebraically averaging near zero), and then a Fourier analysis for the constituent frequency and possibly some harmonics are made of the constituent hourly means. An augmenting factor is applied to the amplitude to correct for the small reduction caused by using solar rather than constituent hours. Because it is impossible to have a finite

length of observations that is commensurable for all tidal frequencies, the length of record chosen for Fourier analysis minimizes but does not completely remove the effect of interfering constituents. This effect is removed to a large extent by a further process called "elimination." In theory, repetitive elimination would refine the results even more, but this was not found necessary.

It is far more direct to solve for all constituents simultaneously, obtaining the complex amplitudes of cosine curves for each constituent that minimize the residuals in a least-square sense. For n constituents, we solve for $2n + 1$ coefficients (including the mean term) and then obtain the harmonic constants (amplitude and phase lag for each constituent) by simple trigonometry. For 37 constituents, a 75×75 matrix could not be solved without very large computers, and hence this approach was unthinkable until recent times. The procedure has been found to be more accurate for one-year analysis than the traditional methods, and it is now used in the National Ocean Survey (NOS) in this country and by the tidal authority in the United Kingdom, the Bidston Observatory of the Institute of Oceanographic Sciences. The process does not require equally spaced data (in the time sense) and will work with data in random time, although the software is more complicated as each data point must be identified in time. Comparative tests of the traditional and least-squares analysis procedures for 29-day series (approximate synodic period for phase, parallax, and declination of the moon) disclosed no advantage in the newer method; therefore the classical procedure has been retained in NOS, but the procedure has been modernized (in particular, stencil summing has been done away with). In 29-day analysis only K_1 and O_1 and M_2, S_2, and N_2 can ordinarily be resolved for species 1 and 2 (1 and 2 cycles per day, respectively).* Because the classical method infers the effect of other disturbing constituents in the elimination process, this probably gives it an advantage over the straightforward least-squares analysis for the five constituents that offsets the improvements implicit in the latter process.

2. Extended harmonic analysis. Once tidal predictions were no longer constrained to a finite set of constituents, the door was opened to improving shallow-water predictions by adding additional compound tides (integral sums and differences of principal constituents). Two independent studies used 114 constituents with frequencies up to 12 cycles per day to improve predictions for Anchorage (Alaska) and for the Thames estuary (Zetler and Cummings, 1967; Rossiter and Lennon, 1968). Solving for 114 constituents simultaneously would involve a 229×229 matrix; fortunately this is not necessary as it has been demonstrated that sidebands are important only within each species (thus K_1 sidebands do not affect M_2), and therefore the solution may be simplified by requiring only that any particular matrix include all constituents within a species. A. S. Franco (Instituto de Pesquisas Tecnológicas, São Paulo, Brazil) has recently developed another approach using a matrix of the Fourier coefficients within a species to solve for constituent harmonic constants including numerous compound tides.

3. Determination of the continuum. On occasion, Walter Munk has used the expression, "Noise exists everywhere but in textbooks on tides." He was referring to the tendency to look only at the tidal lines in the spectrum, disregarding the level of the continuum between the lines. In a plot of energy (or amplitude) versus frequency, the extent to which the value at a tidal line protrudes above the continuum is a measure of reliability of the tidal constants for the constituent. Over a period of a few years, Munk and various associates made a determined effort to evaluate the level of the continuum for a wide band of frequencies. Very sharp filters discriminating against tidal lines were used to establish the continuum between tidal species (Munk and Bullard, 1963). Then it was demonstrated that if the noise level is sufficiently low, two tidal constituents can be resolved in a record of length shorter than their synodic period (Munk and Hasselmann, 1964). An extremely fine-resolution analysis in the low frequencies (0 to 0.75 cycle per day) showed the continuum decreasing monotonically in energy for higher frequencies and no significant peaks other than previously determined tidal constituents (Groves and Zetler, 1964). Finally cusps of energy were found between tidal groups (separations of 1 cycle per month) and even between tidal lines within a group (Munk *et al.*, 1965). The knowledge derived in these studies was particularly important in the development of response analysis and prediction (see below) and serves to establish a limit on the accuracy to which one may aspire for barotropic tide predictions in the open ocean.

4. Response analysis and prediction. For any linear system, an input function $\chi_m(t)$ and an output function $\chi_n(t)$ are releated according to

$$\chi_n(t) = \int_{-\infty}^{\infty} \chi_m(t - \tau) w_{mn}(\tau) d\tau + \text{noise}(t),$$

where $w_{mn}(\tau)$ is the "impulse response" of the system, and its Fourier transform

$$Z_{mn}(f) = \int_{-\infty}^{\infty} w_{mn}(\tau) e^{-2\pi i f \tau} d\tau = R_{mn}(f)\, e^{i\phi_{mn}(f)}$$

is the system admittance (coherent output/input) at frequency f.

Response tidal analysis and prediction (Munk and Cartwright, 1966) uses as input functions the time-variable spherical harmonics of the gravitational potential and of radian flux* on the earth's surface. In practice, the integrals are replaced by summations; χ_m, w, and Z are

*Principal tidal constituents: K_1 is lunisolar diurnal, O_1 is lunar diurnal, M_2 and N_2 are lunar semidiurnal, and S_2 is solar semidiurnal.

*A function designed to vary with the radiant energy falling on a unit surface in a unit time; it is related to daily atmospheric pressure and wind variations and to seasonal changes in ocean temperature.

complex. The discrete set of w values are termed response weights.

This method is the first successful major departure from the traditional solutions in which the tide oscillations are described by the amplitudes and phase lags for a finite set of predetermined frequencies. Subsequently it was recommended that response analysis of short records (about one month) of pelagic tidal measurements use a response prediction at a nearby coastal station as the reference series (Cartwright *et al.*, 1969). The calculation of traditional harmonic constants from response admittances made results of the two methods compatible (Zetler *et al.*, 1969). The optimum number of weights in response analysis depends directly on the length of record and inversely on noise level in a tidal band; more weights degrade the prediction and generate an artificial wiggliness in the admittance (Zetler and Munk, 1975). This study showed for the first time that better results are obtained by centering the lags to a potential retarded by the age of the tide* rather than to a potential centered on the prediction time.

SCOR† Working Group 27, "Tides of the Open Sea," conducted an analysis workshop in conjunction with an intercomparison of open sea tidal pressure sensors. The report (Unesco, 1975) shows a clear superiority for response procedures as compared with classical methods used by various national tidal groups. In addition to this statistical advantage, response analysis is more intellectually pleasing in that one uses the entire tide-producing potential rather than having arbitrarily to choose a finite set of tidal frequencies.

The ability to separate gravitational and radiational contributions to the tide resolves an unsatisfactory aspect of results from classical analysis. If one plots the phase angles or the amplitude admittances (ratio of analyzed to equilibrium amplitudes) from traditional analysis against frequency, one usually finds a sharp bend or even a discontinuity at S_2 (30°/solar hour). It is not reasonable that the oceans should exhibit such abrupt changes, particularly always at the same frequency. The smoothness of the plots obtained for the gravitational admittances in Figure 14.5 (Zetler and Munk, 1975) using response procedures is undoubtedly due to the separation of radiational from gravitational inputs, whereas classical analysis produces combinations of the two.

B. Numerical Modeling

A cotidal chart is one showing an areal distribution of times of high tide for a particular constituent relative to the equilibrium (theoretical) time of high tide for the same constituent. The cotidal lines, labeled in either solar or constituent hours, identify the locus of points at which high tide for the constituent occurs simultaneously. A feature of such maps is a system of nodal points, known as amphidromes, at which there is no vertical rise or fall. Some charts also include corange lines, contours of equal constituent amplitude. Near an amphidrome, corange lines tend to be concentric about the amphidrome with amplitudes increasing with distance from the zero-amplitude point.

Historically, cotidal charts have been based on seaward extrapolation from coastal and island observations supplemented by general knowledge of how tides behave in mathematically described basins. Inasmuch as ocean tides are modified significantly in the continental coastal zones, use of coastal and, even worse, harbor and estuary observations make these ocean projections quite speculative. Nevertheless, until huge electronic computers became available, these empirical efforts were as good as could be done. In retrospect, some of the early cotidal charts have been found to be remarkably good and have served many useful purposes; those of Rollin Harris are outstanding examples of an advanced state of the art at the beginning of this century (Figure 14.6).

In a parallel but quite different effort, hydrodynamicists have traditionally been concerned with solving the tidal equations developed by Laplace in 1775. Because of their complexities, various simplifying assumptions for hypothetical basins (such as flat bottom, boundaries, and stability) have been necessary (Hendershott and Munk, 1970). Although much has been learned in these earlier scientific efforts, the solutions have had little or nothing to do with tides in the real oceans. Realistic calculations

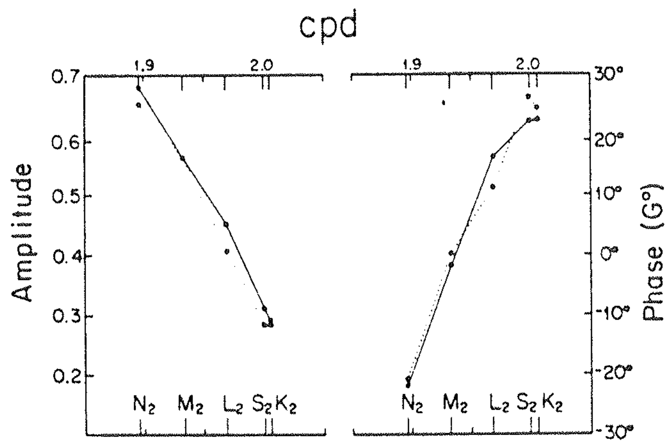

FIGURE 14.5 Amplitude and phase admittances for Bermuda referred to the gravitational potential. The solid lines are grav admittances from (grav and rad) response analysis; the dotted lines are admittances from traditional analysis. The response analysis was for three 355-day series over a 9-year period. The traditional analysis (1 year, 1934, IHB Spec. Pub. 26, #600) furnishes a lumped (vector) sum of grav and rad admittances.

*The time interval between a maximum range in the equilibrium tide and a comparable range at a particular place. For example, equilibrium spring tides occur at new and full moon; in the ocean they occur 0.984 ($S_2° - M_2°$) hours later. There are comparable ages for maximum ranges related to perigee and to maximum declination of the moon.

†Scientific Committee on Ocean Research.

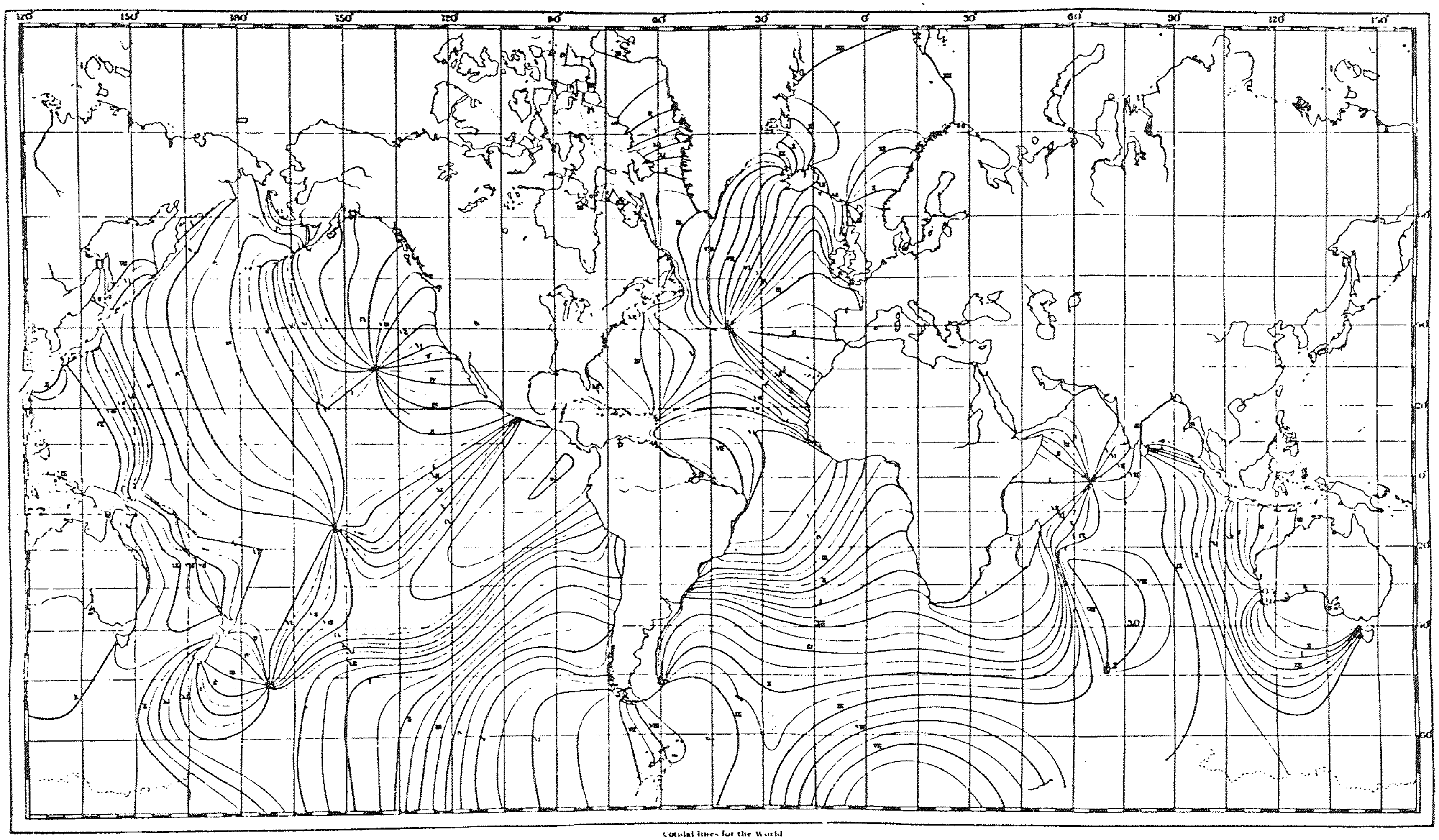

FIGURE 14.6 Harris cotidal chart for M_2.

require detailed bathymetry and boundaries, inconceivable before the advent of large electronic computers and marginal even today. When Pekeris presented a global solution for M_2 at an international meeting about 15 years ago, Joseph Proudman, a prominent tidal authority, commented that he had not anticipated seeing this degree of success in his lifetime.

A recent summary of the activities of SCOR Working Group 27, "Tides of the Open Sea" (Cartwright, 1975) noted various problems in numerical modeling of tides: "There are numerous esoteric sub-problems concerning stability, mesh size, boundary and depth topography, friction, but the most important and difficult appear to be representation of energy losses at the boundaries of shallow seas and at places of steep topography where internal tides are generated, and the solution of the equations modified for a yielding Earth."

Hendershott's more recent numerical studies include energy dissipation, ocean self-attraction, and the solid earth yielding to the weight of the oceanic tidal column. Figure 14.7 is a recent (unpublished) M_2 cotidal chart by Hendershott and Parke. A description of other contemporary numerical tidal solutions (Hendershott, 1973) furnishes information on the tidal frequencies considered, boundary conditions, dissipation mechanisms (if any), and whether earth tides are considered.

Thus, theoretical studies are now using real data in realistic ocean models. Solutions are improving; in particular, it appears that an amphidrome first identified in a numerical study is real. However, results indicate that many ocean areas are nearly resonant at semidiurnal periods. As a result, a variation in mean depth of only several hundred meters may change the M_2 amplitude by a factor of 2 or 3. Furthermore, it has been found that changing the coastline slightly or refining the grid from a 2° to a 1° mesh changed the M_2 amplitude by as much as 3 m (Pekeris and Accad, 1969). This extreme sensitivity makes it difficult to evaluate empirically the contribution of various parameters.

In the past, physical models of various estuaries have been built in order to determine the effect of proposed man-made structures on the tidal regime in the estuary. For example, the model of San Francisco Bay (built in Sausalito by the U.S. Army Corps of Engineers) has been used in studying the effect of proposed salinity barriers on the tide in the Bay. Garrett (1977) recently used a numerical model to study the effect of a proposed power-generating system in the Bay of Fundy, with the large ranges of tide as the energy source. Ordinarily one would expect such a utilization to diminish the tidal range and would need to consider the economic consequences of the anticipated change. However, Garrett found the resonant peak of the basin to be only slightly higher than the M_2 frequency (thus accounting for large tides) and that the proposed structure would reduce the resonant frequency, bringing it even closer to the M_2 frequency, and so would lead to very little, if any, reduction in amplitude.

C. *Recording*

Pelagic pressure sensors, capable of measuring the tide on the sea floor in the deep parts of the oceans are an important technological development in recent years (Unesco, 1975). Pioneered by Frank Snodgrass at Scripps Institution of Oceanography and Marc Eyries in France, the state of the art has rapidly advanced to the point that only financial considerations deter us from obtaining the global grid of tide observations needed for an optimum set of cotidal and corange charts.

In the Unesco report on an international intercomparison of open-sea tidal-pressure sensors, five types of sensor were found to intercalibrate closely (Unesco, 1975). Among the several mooring techniques used in the experiment, the self-contained acoustic "pop-up" unit showed a clear superiority. In the MODE (Mid-Ocean Dynamics Experiment) exercise, tidal data were obtained for the first time from an array of pressure sensors on the sea floor. Figure 14.8 shows that the phase angles obtained for MODE stations match Dietrich's M_2 and K_1 cotidal charts well (Zetler *et al.*, 1975). The M_2 tidal currents inferred from the gradients of observed pressures fit the barotropic component of the observed tidal currents reasonably well; the latter also contain large baroclinic components, whereas the small baroclinic contributions to the pressure gradients have been further reduced because the station spacings are roughly comparable to the baroclinic wavelength.

Numerical modelers have recommended observations at "antiamphidromes" (locations of maximum tidal amplitude) in order to calibrate their results. Furthermore, to work with intermediate scale models (portions of the world's oceans), because of the need for more dense topographic grids, they will need tidal observations at the boundaries of sections in the open ocean. The technology for this is now available, and some programs along these lines are already under way at the Institute of Oceanographic Sciences in England.

Time has passed by the 2000 or so conventional tide gauges used throughout the world; there have been few changes since Lord Kelvin argued the case for using a pencil before the Institution of Civil Engineers in 1882. The gauges continue to measure the height of a float in a stilling well (cylinder with a small orifice near the bottom to filter out wind waves). The only significant change has been the utilization of digital recording on magnetic and punched paper tape; these have significantly reduced tabulation and analysis costs, making it possible to obtain and process more observations with available resources.

14.4 FUTURE RELATED PROGRAMS

SCOR Working Group 27 decided in 1975 that it had achieved its shorter-term objectives and that therefore there was no immediate purpose in its continued existence (Cartwright, 1975). The report indicated that several

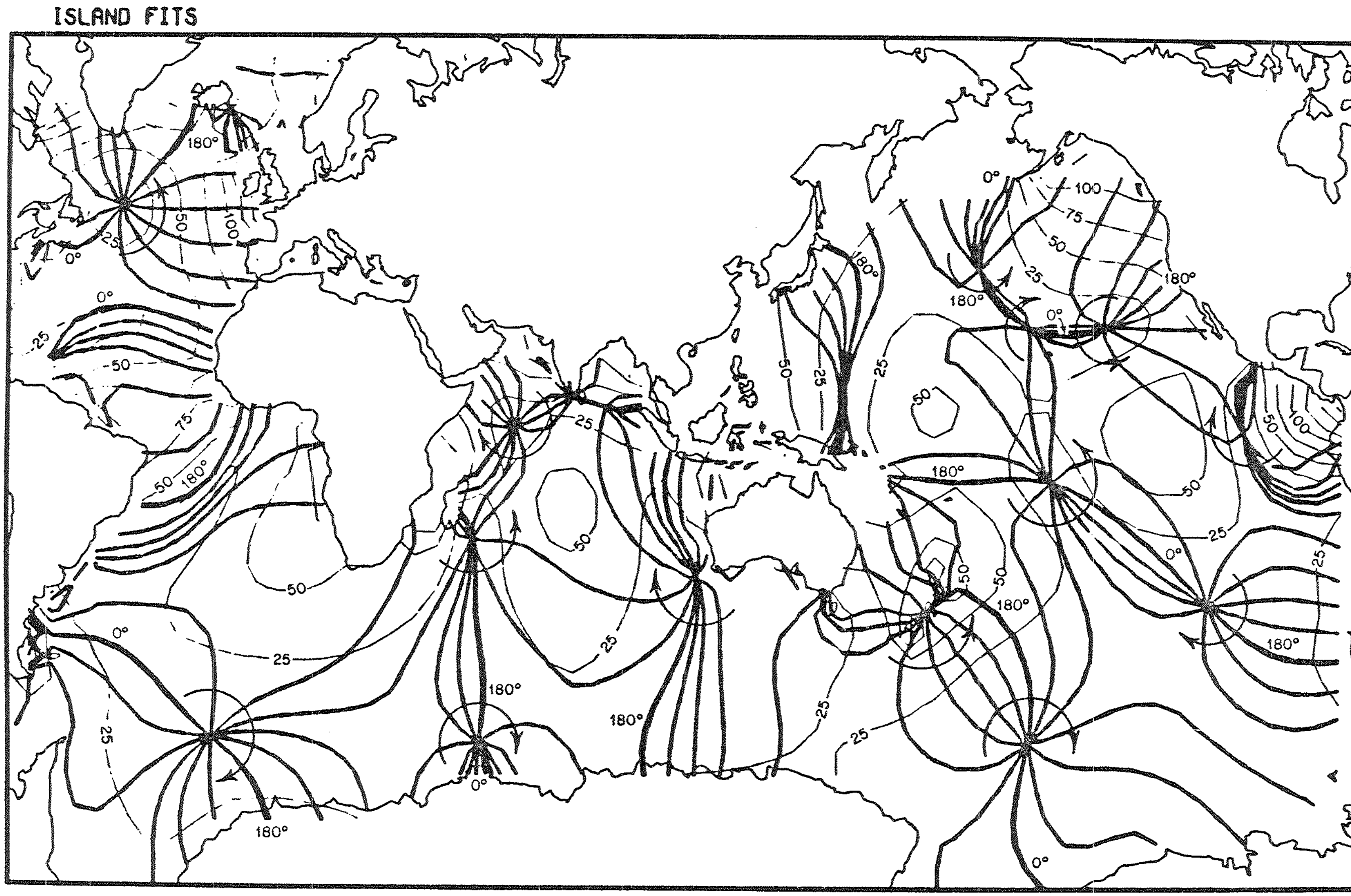

FIGURE 14.7 Hendershott and Parke cotidal and corange chart for M_2.

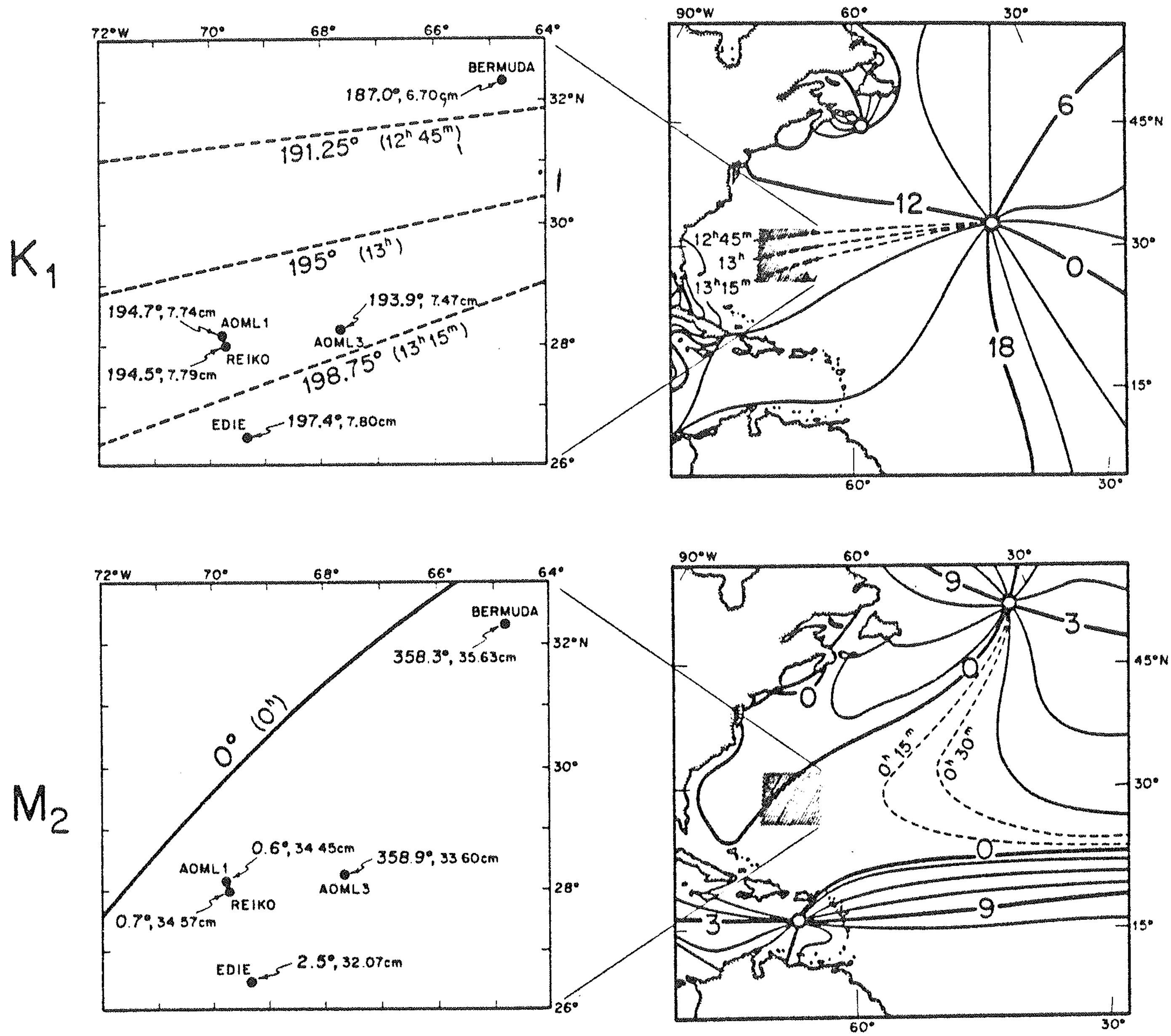

FIGURE 14.8 Right panels show Dietrich's (1944) cotidal lines in the North Atlantic for K_1 and M_2 tides, respectively. Values are in solar hours, with dashed curves designating interpolation by Zetler *et al.* (1975). The MODE area falls within the shaded square, which is shown on an enlarged scale in the left panels, for comparison with the results at MODE stations (●).

types of pelagic pressure sensor have been developed and then shown to be viable and accurate; methods of analysis have compared favorably. Various isolated pelagic observations have been made successfully (possibly as many as 40), but this is a far cry from the Working Group's early global program of some 300 pelagic stations on a 1000-km-square grid. The report suggested that at some future time there might be advantages in organizing working groups coordinating (1) tidal investigations of data from satellite altimetry and (2) studies of baroclinic tides. These and other possible programs are summarized below.

A. *Pelagic Deployments* Although no intensive global program can be anticipated, pelagic tide observations will continue to be made by various organizations. In some cases these will be for local studies, in others to supply boundary data for numerical modeling. For some shelf areas, a few measurements could be used to fit analytical waveforms such as Kelvin and Poincaré waves and thus help to determine the tidal patterns for considerably larger areas (Munk *et al.*, 1970).

The distribution of available data will be improved by two new programs. The Tides and Mean Sea Level Committee (International Association of the Physical Sciences of the Ocean, International Union of Geodesy and Geophysics) plans to publish a loose-leaf inventory of open-sea pressure data including positions, times, publications, and address of authors, and the International Hydrographic Office (Monaco) plans to include tidal constants for pelagic stations in its published lists of harmonic constants.

B. Satellite Data When satellite altimetry and positioning can be made sufficiently accurate, an empirical tidal analysis on a regional or global basis is conceivable. Alternatively, one may have a range of numerical global tide solutions and compute residuals for altimeter profiles along particular tracks to evaluate different model parameters. Orbit definition may be improved by using observations and/or predictions as "sea truth" (for example, tides in the MODE area). There are many complications and obstacles, but tides from satellite altimetry is, we hope, a way of the future.

C. Numerical Models It has been noted earlier that improved geophysical inputs to models are finally leading to results from theory that look somewhat like the real oceans. The proximity in frequency of the ocean tides and of ocean resonances makes the problem rather severe, but, nevertheless, improvements, particularly given more data observed on the sea floor and from space, can be anticipated.

D. Inverse Solutions from Earth-Tide Observations It has been demonstrated that land-based earth tides observed on a gravimeter near an ocean are modified by the ocean tidal loading. It is more controversial, however, whether it is possible to map the open ocean tides by solving the inverse problem using land-based gravity measurements, shore constraints, and a few ocean-bottom stations. Certainly it can be done with an infinity of stations of perfect precision; the number and precision of land and ocean measurements necessary to achieve required accuracy have not yet been determined.

E. Baroclinic Tides Baroclinic tides probably are largely generated from barotropic tides at shelf edges and over steep topographic features. This inadequately explored major oceanographic regime has been estimated to hold 10 to 30 percent of the total global tide energy. In various analyses involving mode separation, there has been no general agreement as to the number of modes necessary, but a small number is generally considered adequate for tides. Observational programs require large numbers of instruments to achieve adequate spatial resolution, and solutions are more difficult because the baroclinic tides are not phase-locked to the tide-producing forces.

Physical oceanographers are keenly interested in the entire spectrum of internal waves, from inertial to Väisälä frequencies. Research, in particular that leading to predictability, has been fruitful at tidal frequencies and, inasmuch as many geophysical problems remain unresolved, it can be anticipated that research in internal tides will continue despite the logistic and theoretical complexities.

14.5 CONCLUSION

With so much of the preceding referring to research by Walter Munk, it seems appropriate to conclude with a quotation from Hilaire Belloc that appeared in a Munk paper: "When they pontificate on the tides it does no great harm, for the sailorman cares nothing for their theories, but goes by real knowledge."

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14.A APPENDIX: AVAILABILITY OF TIDE AND TIDAL CURRENT PREDICTIONS

Tide and tidal current tables are published in advance of each calendar year by the National Ocean Survey, Rockville, Maryland, 20852.

14.A.1 TIDE TABLES

Advance information relative to the rise and fall of the tide is given in annual tide tables. These tables include the predicted times and heights of high and low waters for every day in the year for a number of reference stations and differences for obtaining similar predictions for numerous other places.

Tide Tables, Central and Western Pacific Ocean and Indian Ocean
Tide Tables, East Coast of North and South America (including Greenland)
Tide Tables, Europe and West Coast of Africa (including the Mediterranean Sea)
Tide Tables, West Coast of North and South America (including the Hawaiian Islands)

14.A.2 TIDAL CURRENT TABLES

Accompanying the rise and fall of the tide is a periodic horizontal flow of the water known as the tidal current. Advance information relative to these currents is made available in annual tidal current tables, which include daily predictions of the times of slack water and the times and velocities of strength of flood and ebb currents for a number of waterways together with differences for obtaining predictions for numerous other places.

Tidal Current Tables, Atlantic Coast of North America
Tidal Current Tables, Pacific Coast of North America and Asia

A list of other sales agents appears semiannually in the Notice to Mariners, published weekly by the Defense Mapping Agency, Hydrographic Center, or may be obtained on request from the National Ocean Survey.

Tide and tidal current tables, published by various other maritime nations (usually by the Hydrographic Service), may contain more extensive coverage in home waters. Ordinarily tide tables are prepared for an indicated standard time zone and, when appropriate, can be corrected for local daylight savings time by adding 1 h. Predicted tidal heights are referred to the datum of local large-scale nautical charts.

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